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know, love, serve; God, Ourselves & Others

Mathematics

Subtraction Methods

This booklet covers the methods that we teach the children to use in school. While there are specific methods that we teach in each Year Group the children will have methods that they feel more confident with. Our aim is to develop an understanding of the Maths behind these methods rather than teaching the children steps in a process so talking and explaining these methods is as important as carrying them out.

Other maths material provided by the school includes:

- Creating Mathematicians
- Counting, Addition, Subtraction, Multiplication and Division Methods
- Half termly, Key Instant Recall Facts (KIRFs)
- Topic, Knowledge Organisers

MATHS NO PROBLEM!

PRIMARY MATHS SERIES

SCHEME OF WORK – YEAR 4

For more information on the what is covered in each year group our Schemes of Work are published on our website and include:

- An overview of the national curriculum topics covered during the school year by term
- A full lesson breakdown for each national curriculum topic and the learning objective for each lesson

CPA Approach

Concrete, Pictorial, Abstract (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths in children. Often referred to as the concrete, representational, abstract framework, CPA was developed by American psychologist Jerome Bruner. It is an essential technique within the Singapore method of teaching maths for mastery.

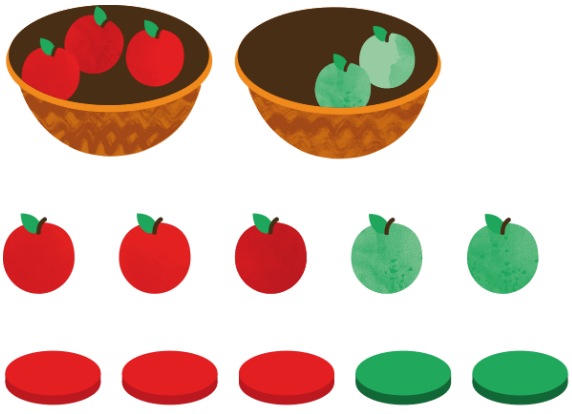
At a glance

- An essential technique of maths mastery that builds on a child's existing understanding
- A highly effective framework for progressing pupils to abstract concepts like fractions
- Involves concrete materials and pictorial/representational diagrams
- Based on research by psychologist Jerome Bruner
- Along with bar modelling and number bonds, it is an essential maths mastery strategy

Background to the CPA framework

Children (and adults!) can find maths difficult because it is abstract. The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems. The CPA framework is so established in Singapore maths teaching that the Ministry of Education will not approve any teaching materials that do not use the approach.

Concrete step of CPA



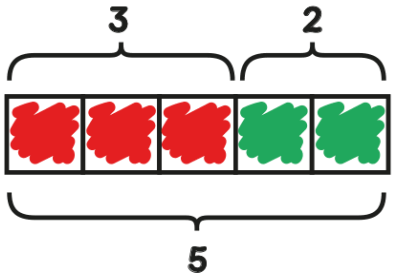
Concrete is the "doing" stage. During this stage, students use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.

For example, if a problem involves adding pieces of fruit, children can first handle actual fruit. From there, they can progress to handling abstract counters or cubes which represent the fruit.

Pictorial step of CPA

Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps students visualise abstract problems and make them more accessible.



Abstract step of CPA

Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, –, x, /) to indicate addition, multiplication or division.

$$3 + 2 = \boxed{5}$$

Making CPA work

Although we've presented CPA as three distinct stages, a skilled teacher will go back and forth between each stage to reinforce concepts.

The MNP Primary Series approach encourages teachers to vary the apparatus that children use in class. For example, students might one day use counters, another day they might use a ten frame. Likewise, children are encouraged to represent the day's maths problem in a variety of ways. For example, drawing an array, a number bond diagram or a bar model.

By systematically varying the apparatus and methods used to solve a problem, children can craft powerful mental connections between the concrete, pictorial, and abstract phases.

When teaching young children numbers, counters and multi-link cubes are more commonly used. However, removing concrete materials exposes children to abstract concepts too early. As a result, they miss out on the opportunity to build a conceptual mathematical understanding that can propel them through their education.

It is important to recognise that the CPA model is a progression. By the end of KS1, children need to be able to go beyond the use of concrete equipment to access learning using either pictorial representations or abstract understanding. What is important, therefore, is that all learners, however young, can see the connections between each representation.

Contents

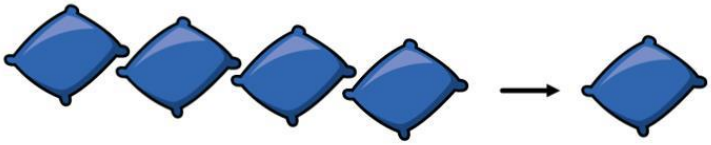
Subtraction Methods

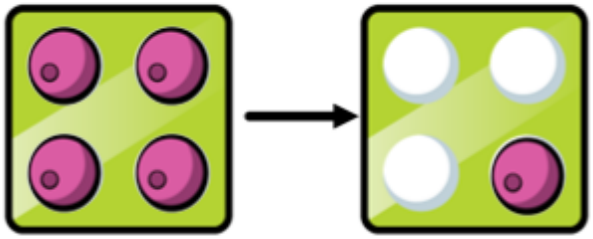
Subtraction by Crossing Out

Year 1

Children start to learn about subtraction by physically taking away and removing objects from a whole. In class this is done with actually objects or with maths equipment that represent what is happening.

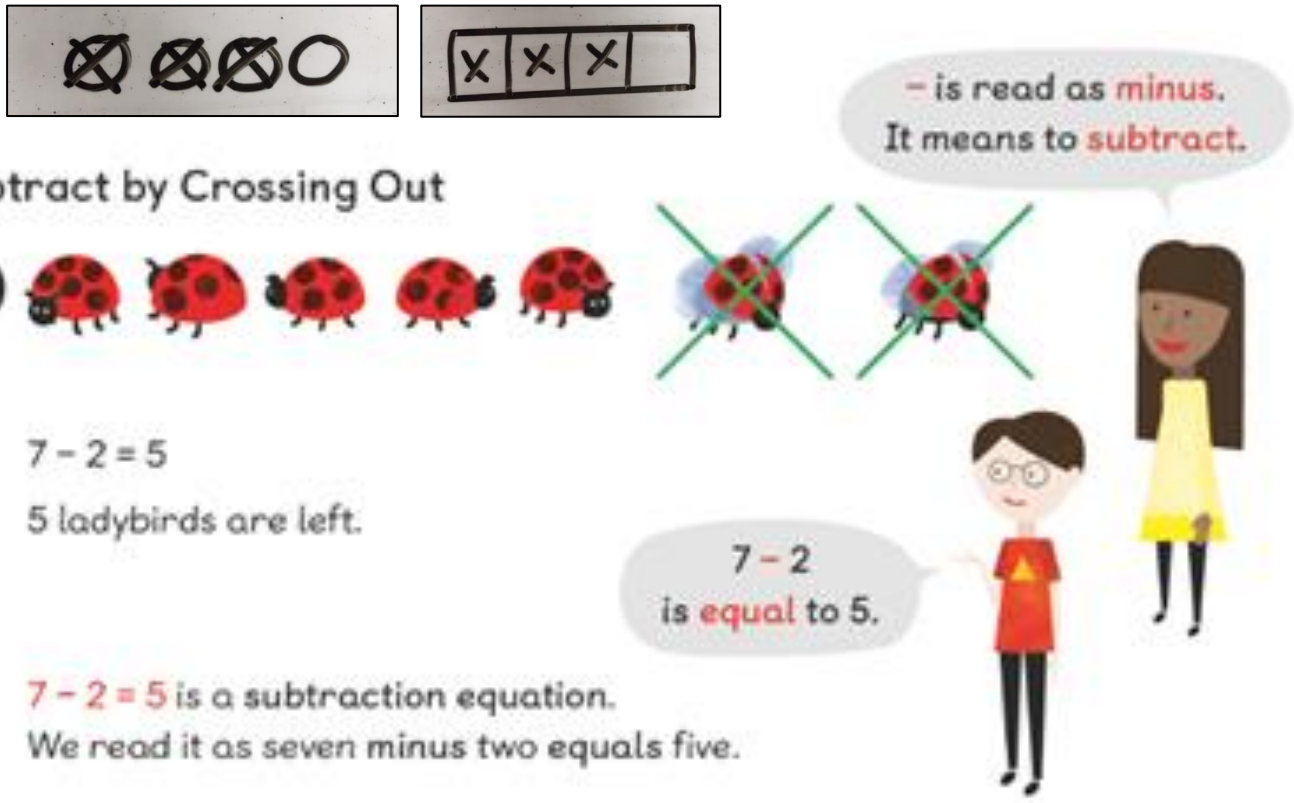
For example, if I have 4 beanbags and take away 3 the children can do this physically and end up with 1 beanbag.





Using a 4 Numicon piece and some pegs (in each hole) I can represent what is happening with the equipment.

Children can move on from concrete examples to pictorial using either pictures provided or diagrams that they draw themselves.

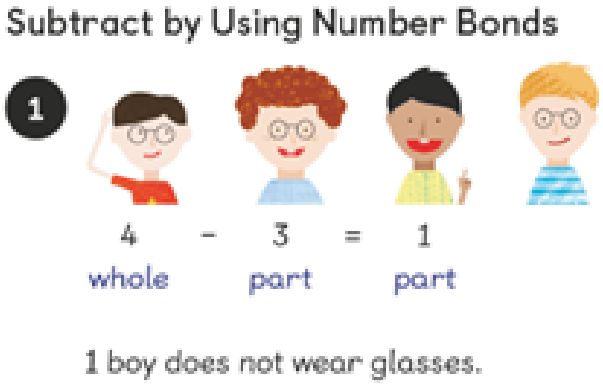


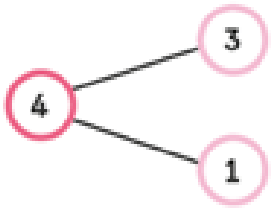
It is important that the children practise saying what is happening and using a range of vocabulary (as above) to explain how all methods work.

Subtraction using Number Bonds & Number Facts

Year 1

Children learn that a number is made up of other numbers. They find as many ways as possible to construct numbers. This is fundamental to all Mathematics as later methods taught require the children to be able to regroup numbers (subtraction) and rename (addition). This also helps the children develop quick mental strategies by making numbers that are easy to use (10s, 100s, etc)



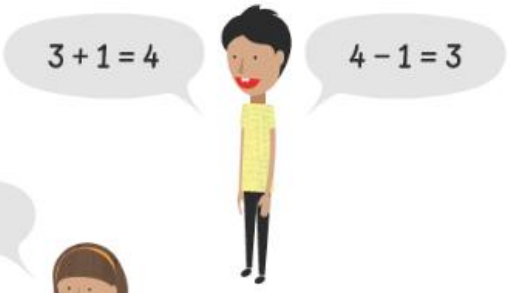


This is the Part Whole Model and is used through maths. In this case there are two parts, but you can break a whole into as many parts as you need.

Year 2

Children are taught families of addition and subtraction facts. The more secure they are with these the better they will be at applying them to new situations.

She has 3 baskets left after giving away 1 basket of apples.



She has 30 apples left after giving away 10 apples.

It is important that children can work with whole tens just as easily as working with ones.

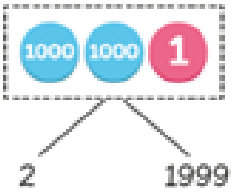
Subtraction using Number Bonds & Number Facts

Year 3

Year 4

Children use strategies and patterns learnt in previous years with increasingly large numbers. Number bonds to whole tens, hundreds and thousands are increasingly important as is being able to regroup numbers.

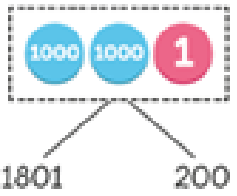
1 2001 – 189



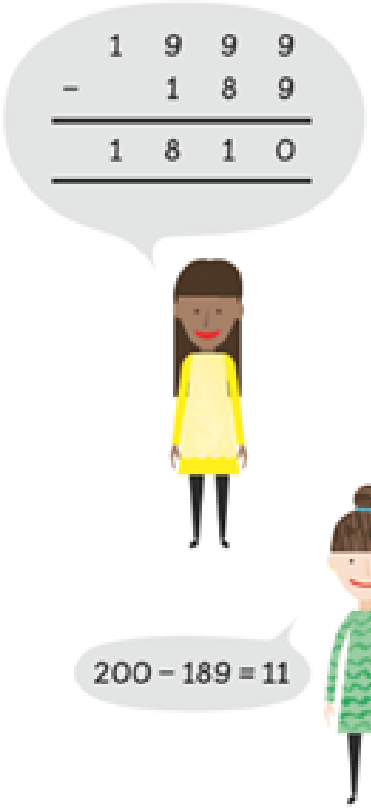
1999 – 189 = 1810

2001 – 189 = 1812

2 2001 – 189



2001 – 189 = 1801 + 11 = 1812



Done as a written method there are lots of opportunities for mistakes in 2001-189 however knowing that 2001 is the same as 1999 + 2 (regrouping) allows for a much easier subtraction.

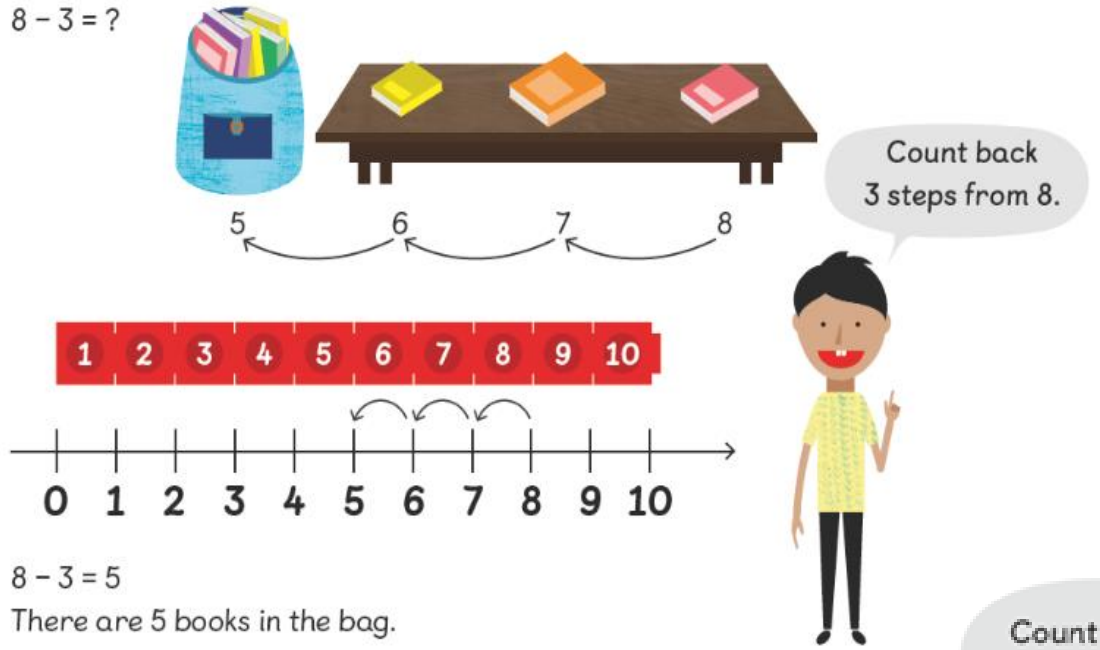
Regrouping can be done in any number of ways here 2001 is regrouped to be:

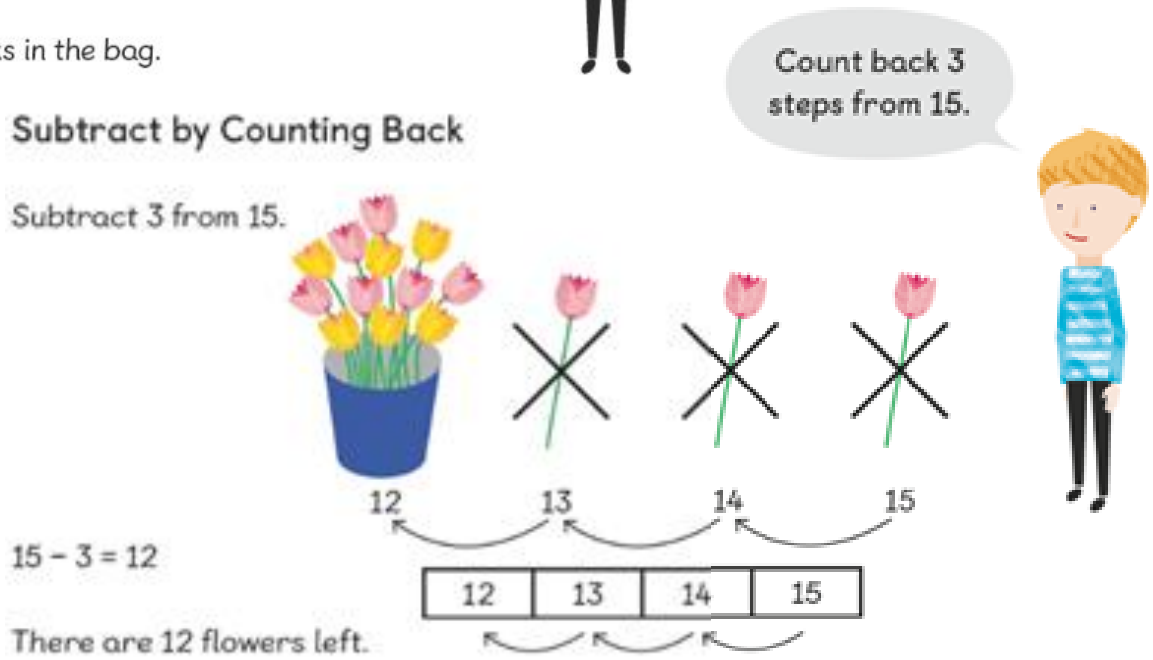
1801 + 200

Subtraction by Counting Back

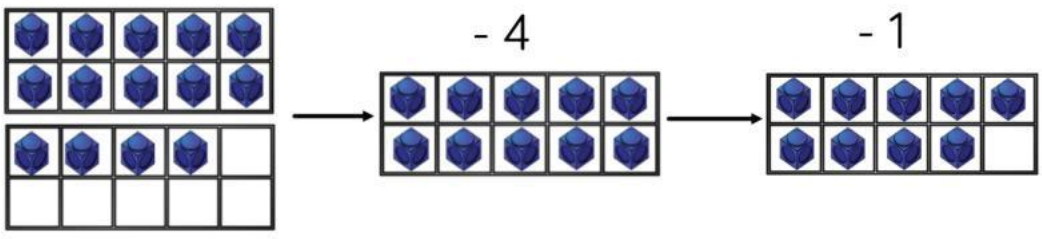
Year 1

Children are taught to count back using a wide range of equipment and compare it to how it would be represented by different models, including number lines, number tracks, Numicon, multilink cubes and tens frames.





14 – 5 using a ten frame

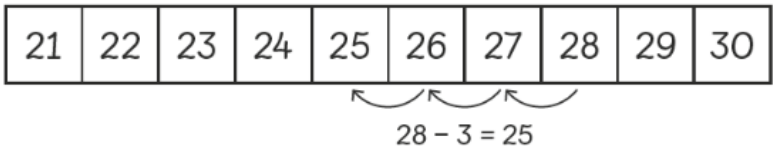


Subtraction by Counting Back

Year 2

Subtract 3 from 28.

Method 1 Count back from 28.



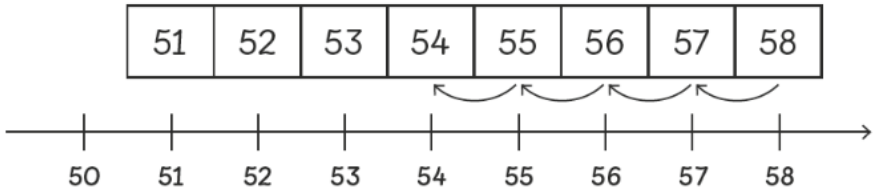
28 − 3 = 25

Year 3

Counting back in Ones

Subtract 4 from 58.

Method 1 Count back from 58.



58 − 4 = 54

Counting back in Tens

Subtract 40 from 658.

Method 1 Count back in tens from 658.

658 − 40 = 618

Counting back in Hundreds

Subtract 500 from 658.

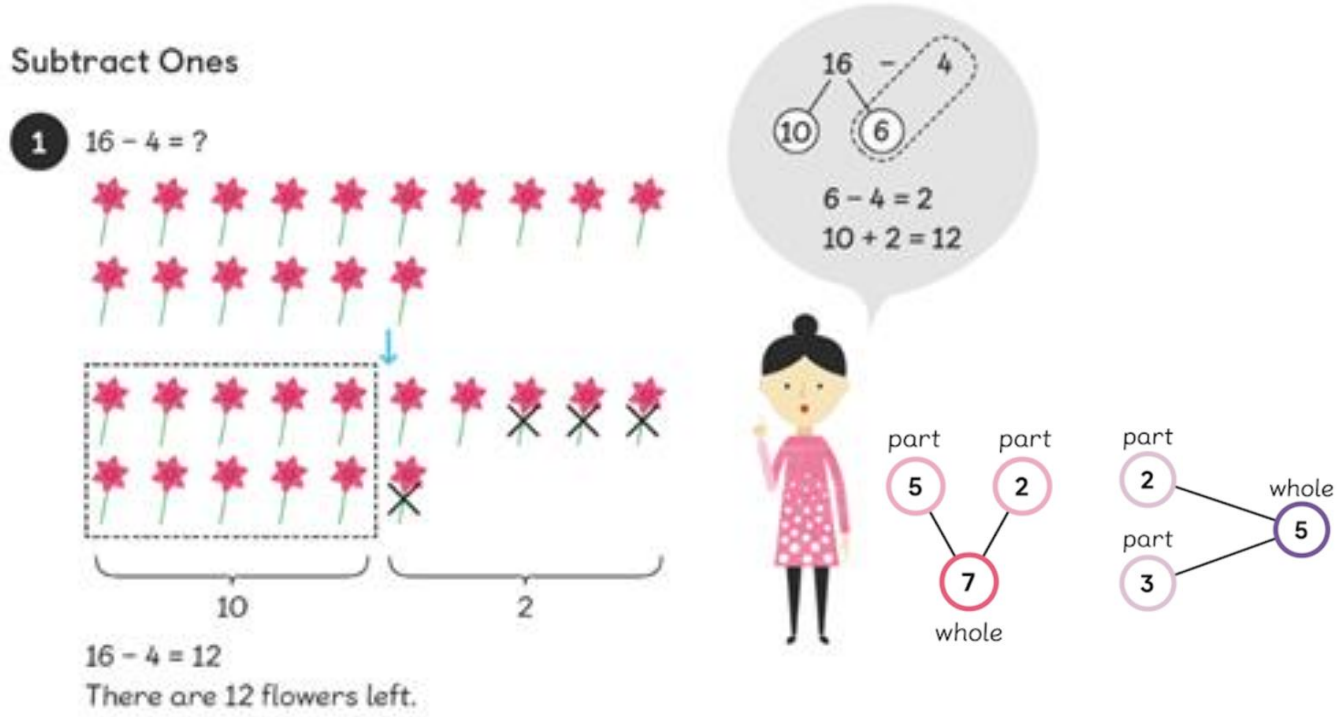
Method 1 Count back in hundreds from 658.

658 − 500 = 158

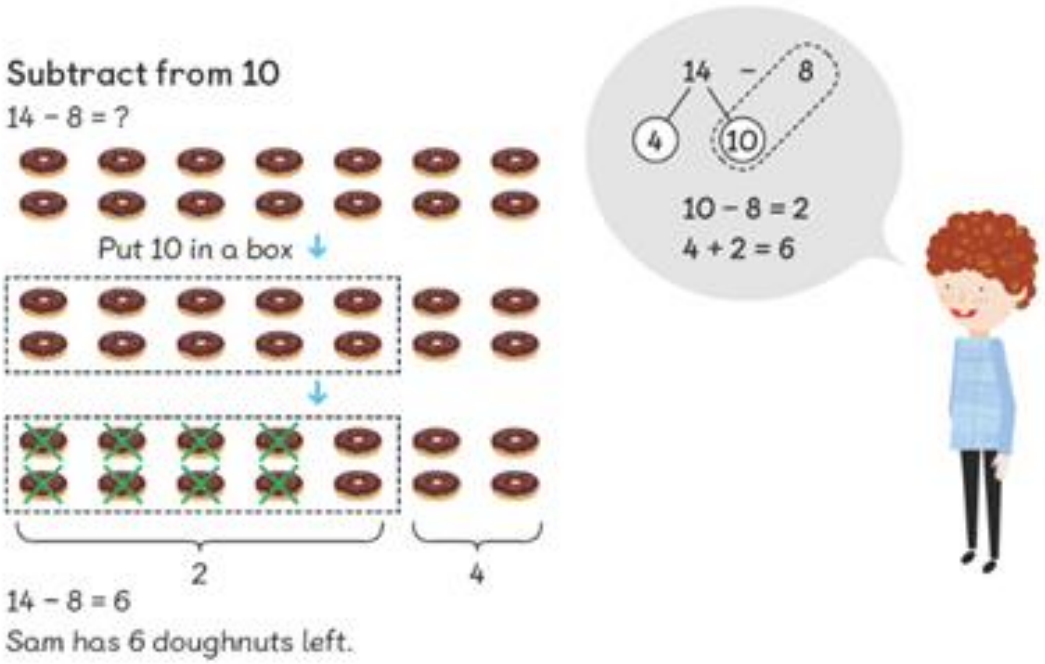
Subtracting Ones, Tens, Hundreds, …

Year 1

Subtracting Ones



Children use the Part Whole Model to help them organise their thinking and explain what they are doing.



Subtraction from ten can often be an easier strategy for the children to use.

Year 2

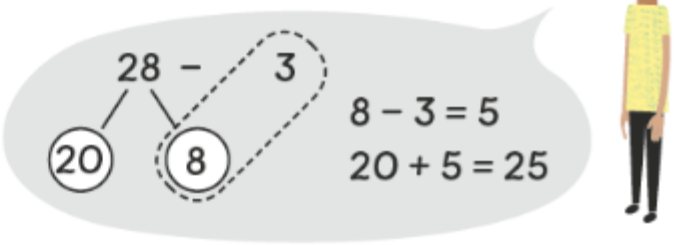
Subtracting Ones

Children are encouraged to talk about their methods and show using concrete objects.

Method 2 Subtract ones.

2 tens 5 ones

28 − 3 = 25



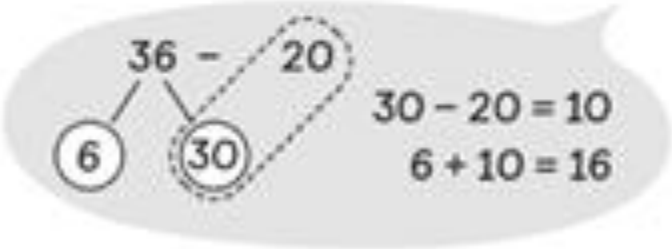
Subtracting Tens

Subtract 20 from 36.

Method 1 Count back in tens from 36.

36 − 20 = 16

Method 2 Subtract tens.

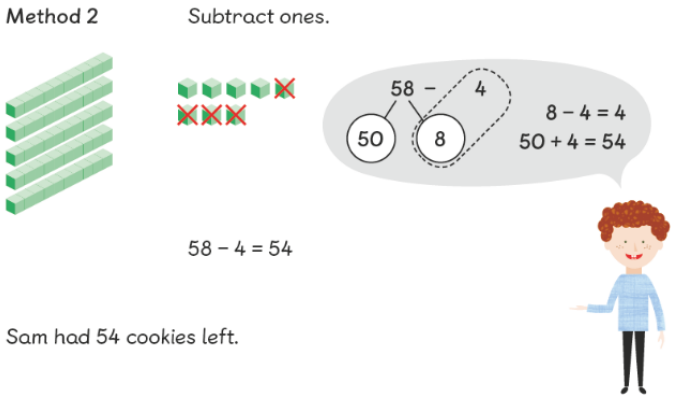


36 − 20 = 16

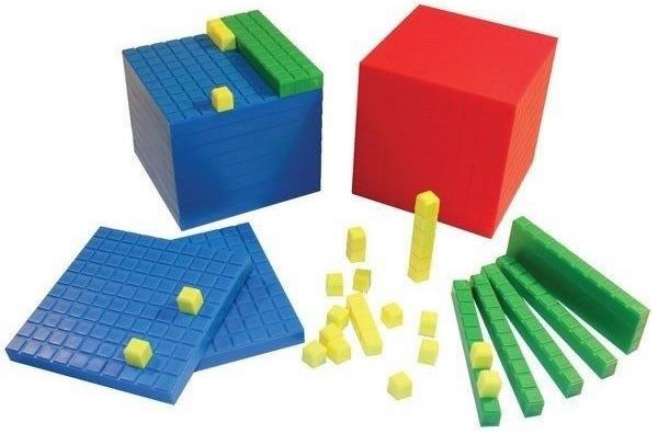
Subtracting Ones, Tens, Hundreds, …

Year 3

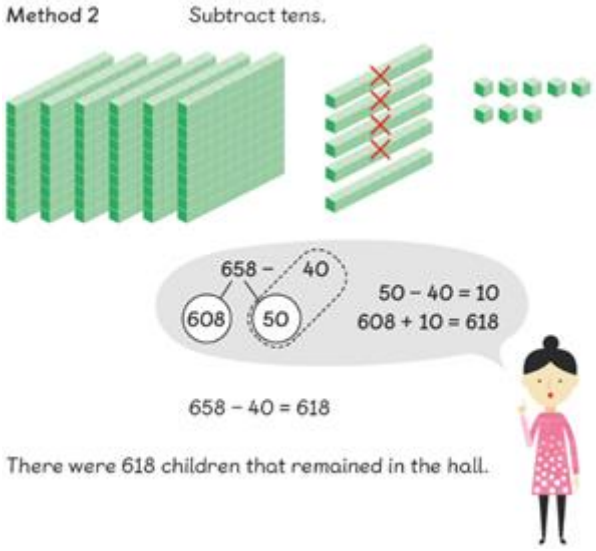
Subtraction in Ones



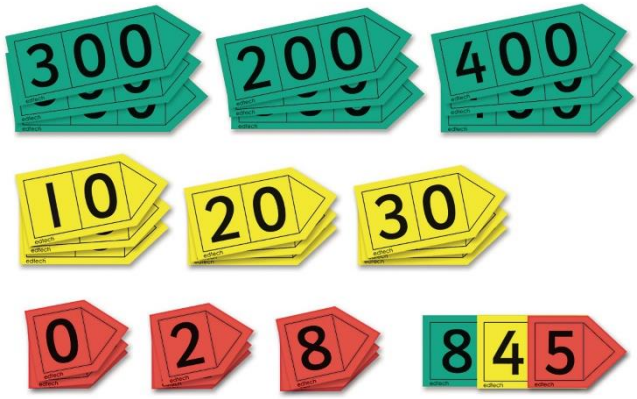
Children use Dienes or Base Ten Blocks to develop their understanding of larger numbers and the place value of each digit.



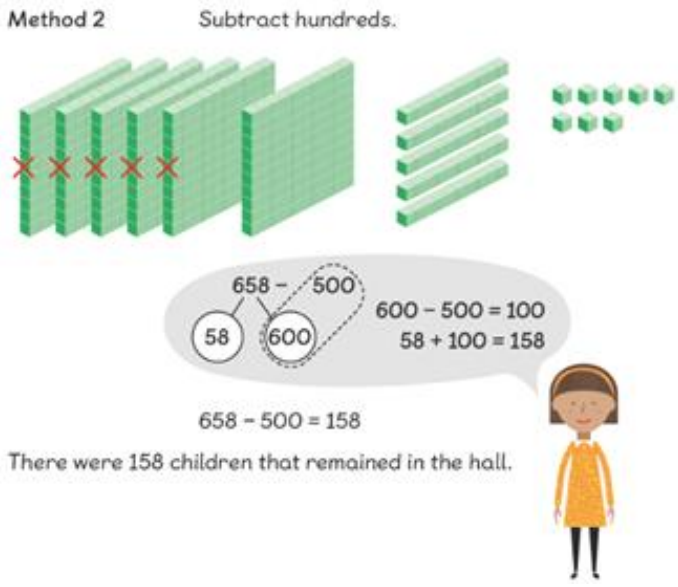
Subtraction in Tens



Place value cards also help.



Subtraction in Hundreds



Initially subtraction will not require Regrouping (previous referred to as borrowing or stealing)

Year 1

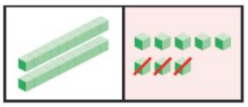
Year 2

Taking Ones

Method 3 Use  to subtract.

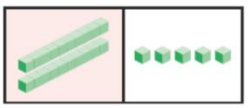
Step 1 Subtract the ones.

8 ones − 3 ones = 5 ones



	tens	ones
	2	8
−		3
		5

Step 2 Subtract the tens.



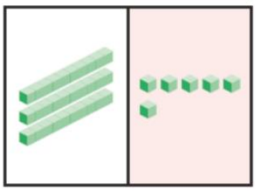
	tens	ones
	2	8
−		3
	2	5

28 − 3 = 25

Taking Whole Tens

Method 3 Use  to subtract.

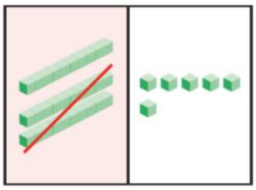
Step 1 Subtract the ones.



	tens	ones
	3	6
−	2	0
		6

Step 2 Subtract the tens.

3 tens − 2 tens = 1 ten



	tens	ones
	3	6
−	2	0
	1	6

36 − 20 = 16

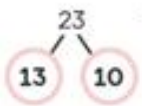
Children start to learn the formal method for subtraction by focusing on changing one place value column at a time, taking either ones or whole tens.

Subtraction with Regrouping

Regrouping is needed when the digit subtraction would result in a negative number. In this example, the issue is in the ones column:

3 – 5 = -2

Method 1 Subtract 5 from 10.



23 − 5 = 18

10 − 5 = 5

5 + 13 = 18

	tens	ones
	2	3
−		5
	2	2

I subtract 3 ones from 5 ones.

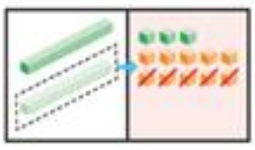
Hannah is wrong. Why?

Method 2 Use  to subtract.

Step 1 Regroup 1 ten into 10 ones.

Subtract the ones.

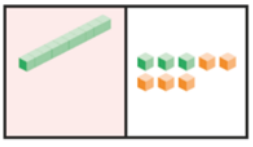
13 ones − 5 ones = 8 ones



	tens	ones
	~~1~~ 2	13 ~~3~~
−		5
		8

13 − 5 = 8

Step 2 Subtract the tens.



23 − 5 = 18

	tens	ones
	~~1~~ 2	13 ~~3~~
−		5
	1	8

10 − 0 = 10

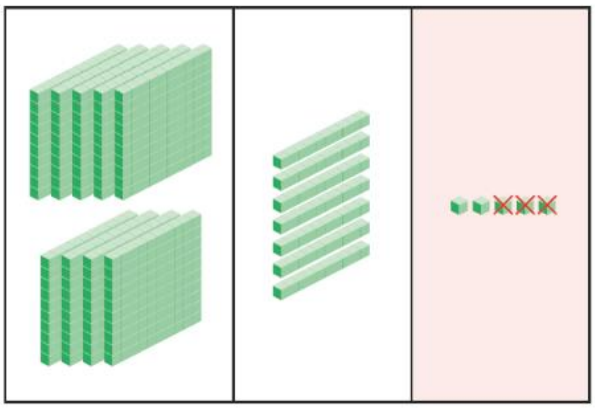
Formal Methods

Year 3

Subtraction with no Regrouping

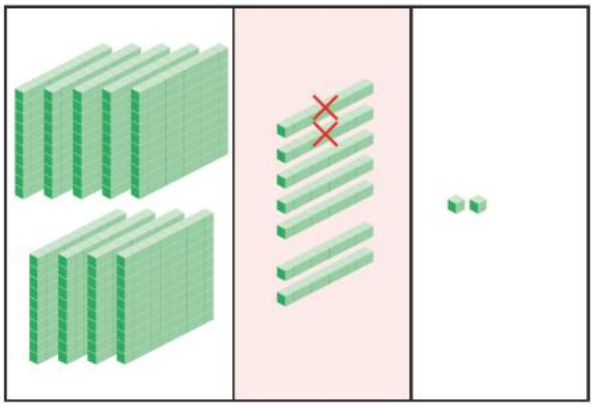
Subtract 723 from 975.

Step 1 Subtract the ones.
5 ones − 3 ones = 2 ones



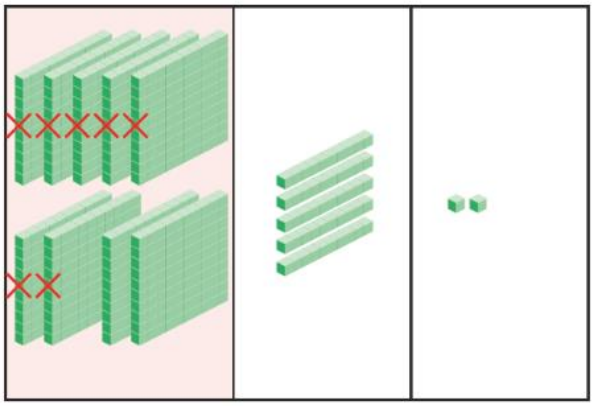
	h	t	o
	9	7	5
−	7	2	3
			2

Step 2 Subtract the tens.
7 tens − 2 tens = 5 tens



	h	t	o
	9	7	5
−	7	2	3
		5	2

Step 3 Subtract the hundreds.
9 hundreds − 7 hundreds = 2 hundreds



	h	t	o
	9	7	5
−	7	2	3
	2	5	2

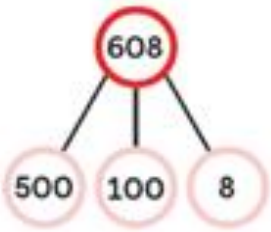
975 − 723 = 252

There were 252 beads left in the jar.

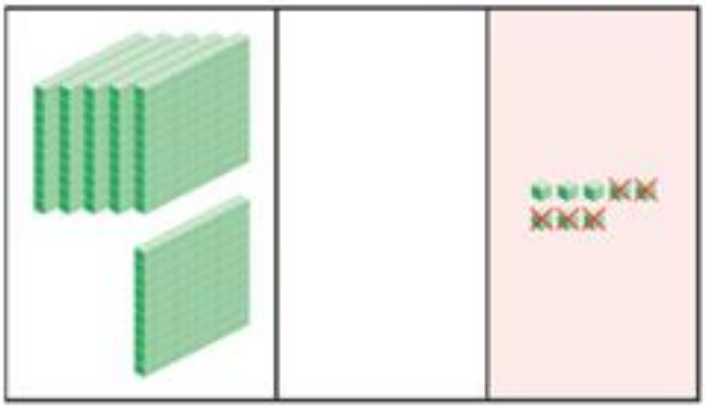
Year 3

Subtraction with Regrouping

Subtract 135 from 608.

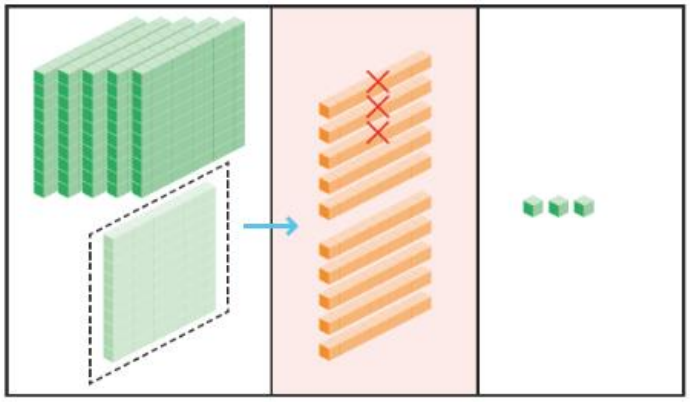


Step 1 Subtract the ones.
8 ones − 5 ones = 3 ones



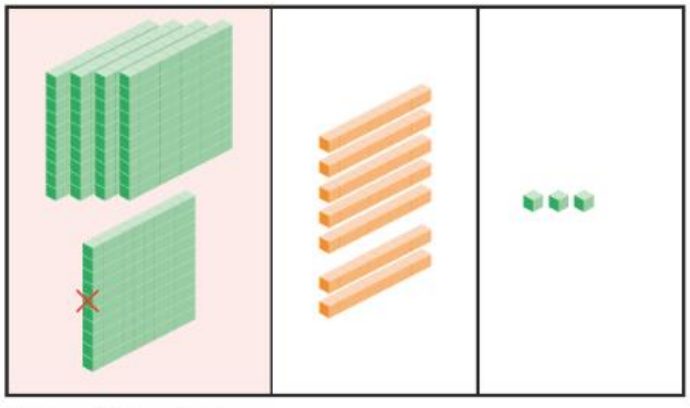
	h	t	o
	6	0	8
−	1	3	5
			3

Step 2 Regroup 1 hundred into 10 tens.
Subtract the tens.
10 tens − 3 tens = 7 tens



	h	t	o
	~~5~~ 6	~~10~~ 0	8
−	1	3	5
		7	3

Step 3 Subtract the hundreds.
5 hundreds − 1 hundred = 4 hundreds

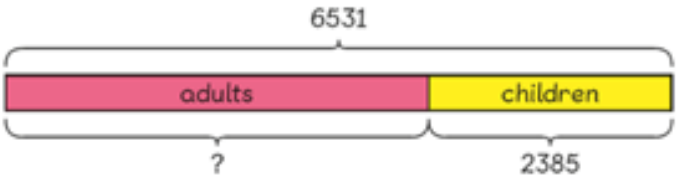


	h	t	o
	~~5~~ 6	~~10~~ 0	8
−	1	3	5
	4	7	3

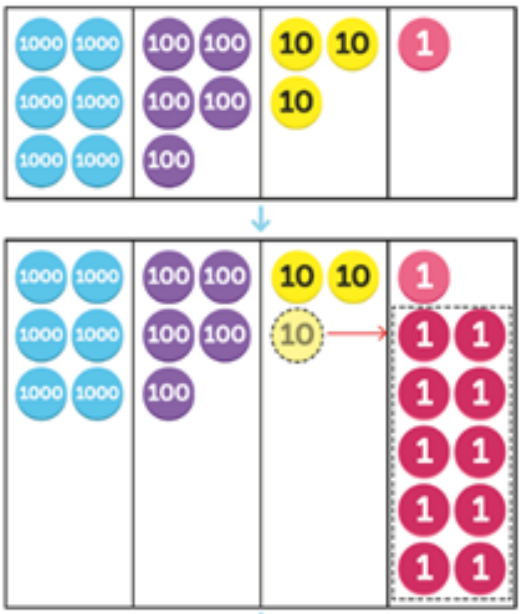
608 − 135 = 473

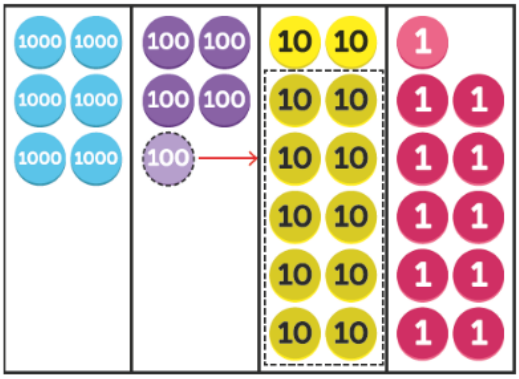
Formal Methods

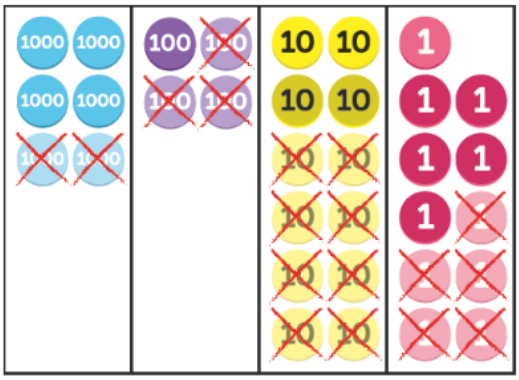
Year 4



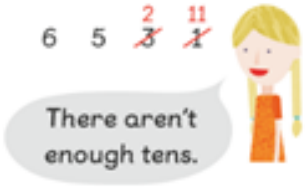
Subtract 2385 from 6531.

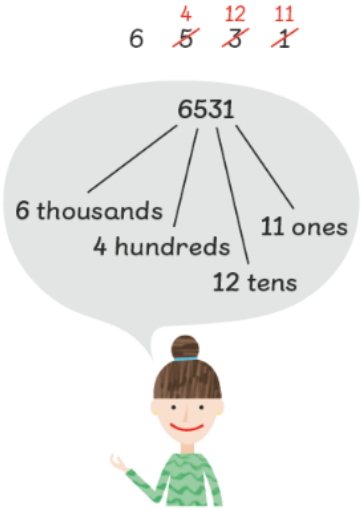






Here place value counters or number discs provide concrete examples for the children to develop their understanding.





$$\begin{array}{r} {\scriptstyle 4\ \ 12\ \ 11} \\ 6\ \ \not{5}\ \ \not{3}\ \ \not{1} \\ -\ 2\ 3\ 8\ 5 \\ \hline 4\ 1\ 4\ 6 \end{array}$$

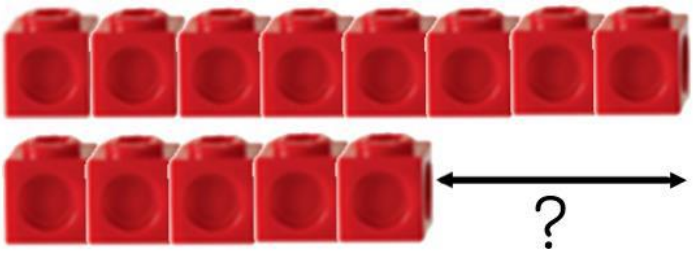
6531 − 2385 = 4146

4146 adults signed up.

Finding the Difference

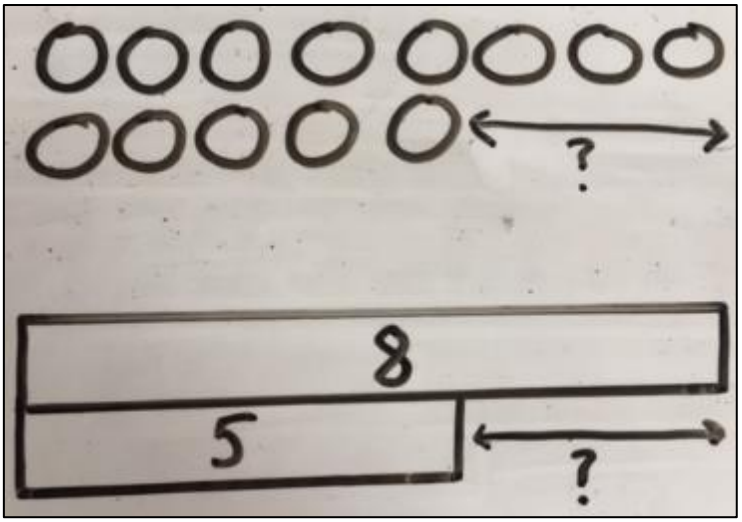
Year 2

Subtraction can be easier or harder depending on whether you look at particular questions as counting back or **finding the difference** and counting on.



Here finding the difference between 8 and 5 is quicker by counting on from 5 to 8 rather than counting back.

Children are taught to make their own pictorial representations of the concrete examples that they work on.



Methods for counting on can be found in the Addition booklet and include Adding using Number Bonds & Number Facts, Counting on using Numicon, multilink cubes, number tracks and lines as well as using the objects themselves.

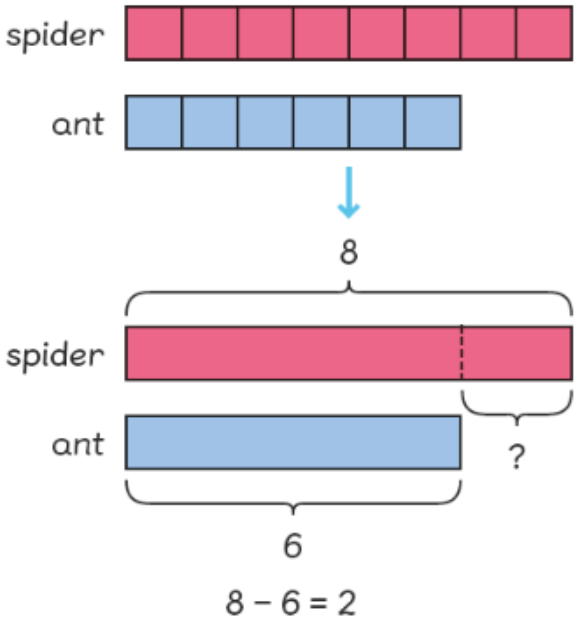
It is important that the children can count on in ones and whole tens.

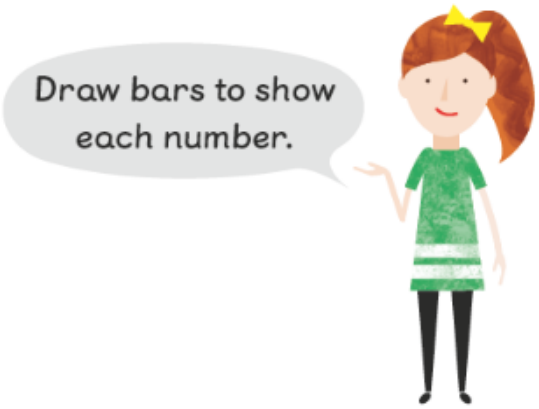
Finding the Difference

Year 3

Children will be shown bar models on a regular basis as a pictorial representation of the problem. While this does not solve the problem, it will help the children understand what is being asked so that they can choose a method.

1 A spider has 8 legs.
An ant has 6 legs.

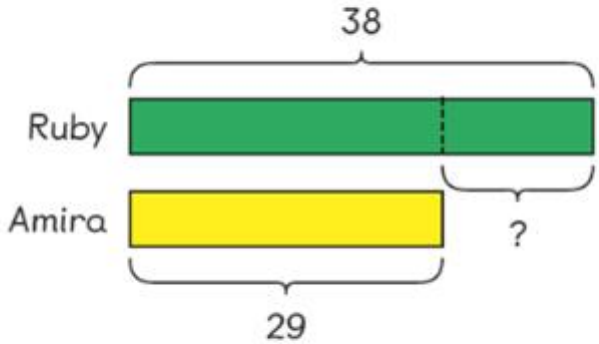




$8 - 6 = 2$

A spider has 2 more legs than an ant.

2 Ruby has 38 stickers.
Amira has 29 stickers.
How many more stickers does Ruby have than Amira?

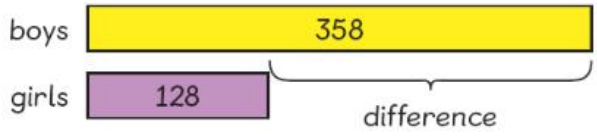


Ruby has  more stickers than Amira.

Children are taught how to choose between the two methods.

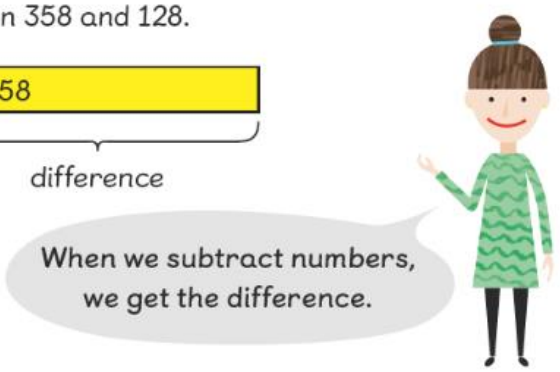
Year 4

1 Find the difference between 358 and 128.



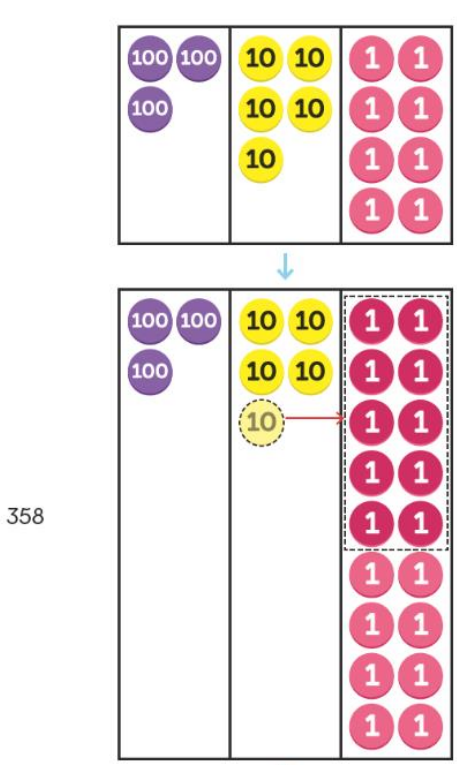
358 − 128 = ☐

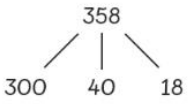
Use base-ten blocks

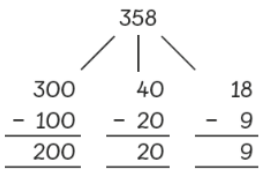


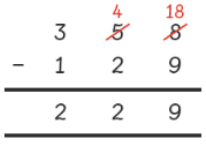
2 Find the difference between 358 and 129.

Here the difference is being worked out, but it is a subtraction rather than counting on.









358 − 129 = 229

The difference between 358 and 129 is 229.