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know, love, serve; God, Ourselves & Others

Mathematics

Multiplication Methods

This booklet covers the methods that we teach the children to use in school. While there are specific methods that we teach in each Year Group the children will have methods that they feel more confident with. Our aim is to develop an understanding of the Maths behind these methods rather than teaching the children steps in a process so talking and explaining these methods is as important as carrying them out.

Other maths material provided by the school includes:

- Creating Mathematicians
- Counting, Addition, Subtraction, Multiplication and Division Methods
- Half termly, Key Instant Recall Facts (KIRFs)
- Topic, Knowledge Organisers

MATHS NO PROBLEM!

PRIMARY MATHS SERIES

SCHEME OF WORK – YEAR 4

For more information on the what is covered in each year group our Schemes of Work are published on our website and include:

- An overview of the national curriculum topics covered during the school year by term
- A full lesson breakdown for each national curriculum topic and the learning objective for each lesson

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CPA Approach

Concrete, Pictorial, Abstract (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths in children. Often referred to as the concrete, representational, abstract framework, CPA was developed by American psychologist Jerome Bruner. It is an essential technique within the Singapore method of teaching maths for mastery.

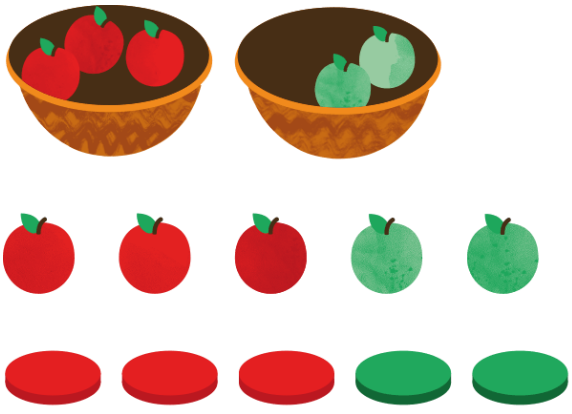
At a glance

- An essential technique of maths mastery that builds on a child's existing understanding
- A highly effective framework for progressing pupils to abstract concepts like fractions
- Involves concrete materials and pictorial/representational diagrams
- Based on research by psychologist Jerome Bruner
- Along with bar modelling and number bonds, it is an essential maths mastery strategy

Background to the CPA framework

Children (and adults!) can find maths difficult because it is abstract. The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems. The CPA framework is so established in Singapore maths teaching that the Ministry of Education will not approve any teaching materials that do not use the approach.

Concrete step of CPA



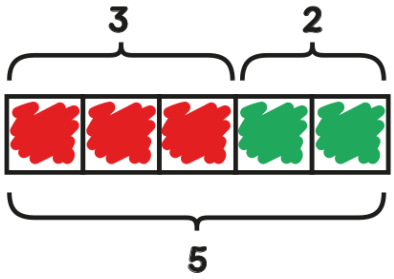
Concrete is the "doing" stage. During this stage, students use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.

For example, if a problem involves adding pieces of fruit, children can first handle actual fruit. From there, they can progress to handling abstract counters or cubes which represent the fruit.

Pictorial step of CPA

Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps students visualise abstract problems and make them more accessible.



Abstract step of CPA

Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, –, x, /) to indicate addition, multiplication or division.

$$3 + 2 = \boxed{5}$$

Making CPA work

Although we've presented CPA as three distinct stages, a skilled teacher will go back and forth between each stage to reinforce concepts.

The MNP Primary Series approach encourages teachers to vary the apparatus that children use in class. For example, students might one day use counters, another day they might use a ten frame. Likewise, children are encouraged to represent the day's maths problem in a variety of ways. For example, drawing an array, a number bond diagram or a bar model.

By systematically varying the apparatus and methods used to solve a problem, children can craft powerful mental connections between the concrete, pictorial, and abstract phases.

When teaching young children numbers, counters and multi-link cubes are more commonly used. However, removing concrete materials exposes children to abstract concepts too early. As a result, they miss out on the opportunity to build a conceptual mathematical understanding that can propel them through their education.

It is important to recognise that the CPA model is a progression. By the end of KS1, children need to be able to go beyond the use of concrete equipment to access learning using either pictorial representations or abstract understanding. What is important, therefore, is that all learners, however young, can see the connections between each representation.

Adding Equal Groups

Year 1

There are 4 trays.

Each tray has 5 .

5, 10, 15, 20

4 trays of 5 = 20
4 groups of 5 = 20
4 fives = 20

There are 20 altogether.

Children will learn that multiplication is groups or lots of something. Initially this will start with concrete objects and they will learn that a method of multiplication is repeated addition. Alongside this they will practise counting in 2s, 5s and 10s.

Year 2

Use in place of a sausage.

Use to make groups of 2.

$1 \times 2 = 2$

$2 \times 2 = 4$

$3 \times 2 = 6$

$4 \times 2 = 8$

Children will learn their:

2, 5 and 10

times tables to solve some multiplication problems and develop an understanding of how these link to addition of equal groups.

How many cupcakes are there altogether?

$4 \times 3 = 12$ is read as 4 times 3 equals 12.

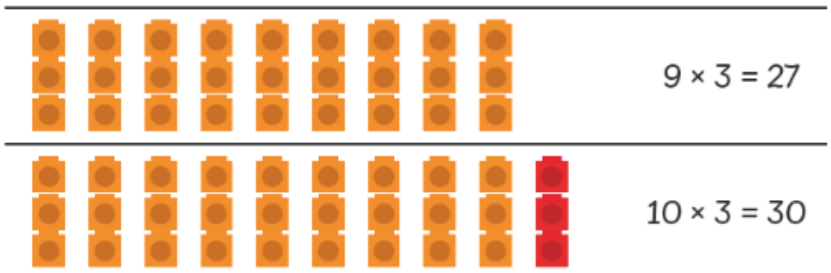
There are 4 groups. Each group has 3 cupcakes.

$3 + 3 + 3 + 3 = 12$
4 threes = 12
4 groups of 3 = 12
$4 \times 3 = 12$

There are 12 cupcakes altogether.

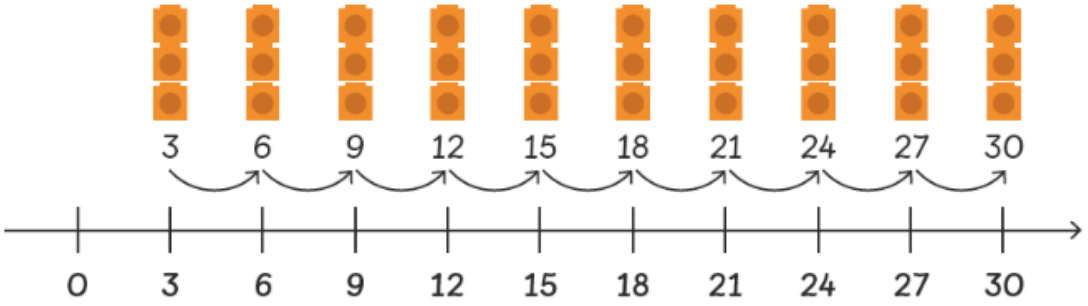
Year 3

Children will continue multiplying by 3, 4 and 8



Learning tables…

Count in threes.

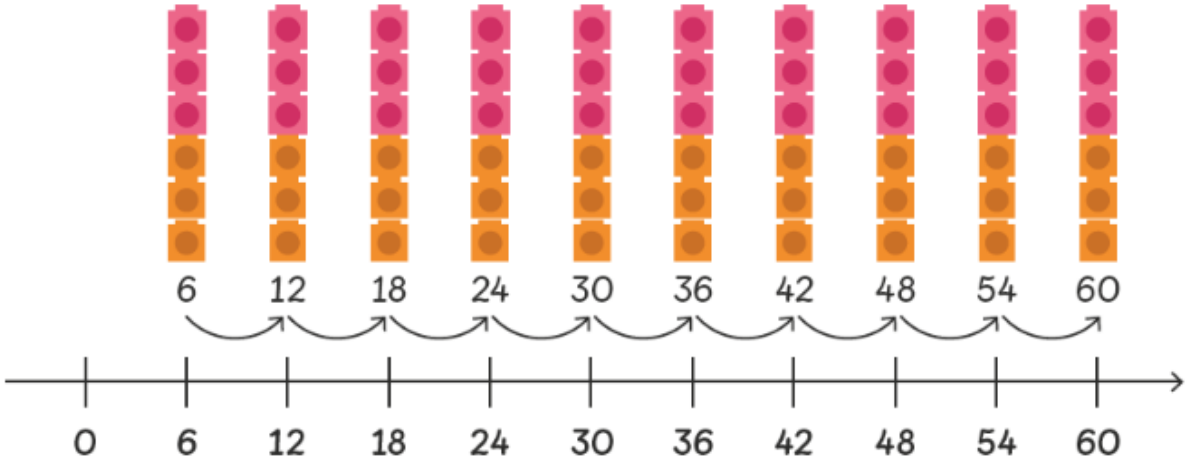


… and linking this to repeated addition.

Year 4

Children will continue multiplying by 6, 7, 9, 11 and 12 making links to previous learning wherever possible. For example the six times table is twice the three times table.

Count in sixes.



Using known Number Facts

Year 3

Multiplying by 3, 4 and 8

1 × 4 = 4	1 × 8 = 8
2 × 4 = 8	2 × 8 = ☐
3 × 4 = 12	3 × 8 = ☐
4 × 4 = 16	4 × 8 = ☐
5 × 4 = 20	5 × 8 = ☐
6 × 4 = 24	6 × 8 = ☐
7 × 4 = 28	7 × 8 = ☐
8 × 4 = 32	8 × 8 = ☐
9 × 4 = 36	9 × 8 = ☐
10 × 4 = 40	10 × 8 = ☐

Year 4

Multiplying by 6, 7, 9, 11 and 12

7 × 300 = ☐

Method 1

300
300
300
300
300
300
+ 300
2100

Method 2

7 × 3 = 21
7 × 3 hundreds = 21 hundreds
7 × 300 = 2100

Method 3

7 × 300 = 7 × 3 × 100
= 7 × 3 × 100
= 21 × 100
= 21 hundreds
= 2100

Making Equal Rows

Year 1

Clever counting involves arranging objects or pictures to make counting easier or highlight a pattern. Arranging the objects in this way prepares the children for working with arrays.

Year 2

$5 \times 2 = 10$ $2 \times 5 = 10$

5×2 is equal to 2×5.

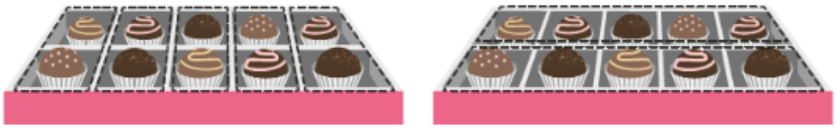
Multiplication and division are inverse operations. Right from the start children are taught these as related operations. There are four number sentences (two using x and two using ÷ which can be written to express the relationship between 2 and 5 and 10.

$2 \times 5 = 10$ $5 \times 2 = 10$ $10 \div 5 = 2$ $10 \div 2 = 5$

Making Arrays

Year 2

Multiplying by 2, 5 and 10

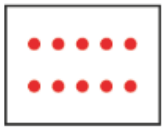


5 × 2 = 10

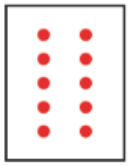
2 × 5 = 10

5 × 2 is equal to 2 × 5.

How many dots are there?



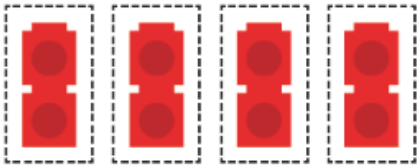
2 × 5 = 10



5 × 2 = 10

Working with arrays allows the children to see that multiplication can be done in either order.

Multiply 2 by 4.

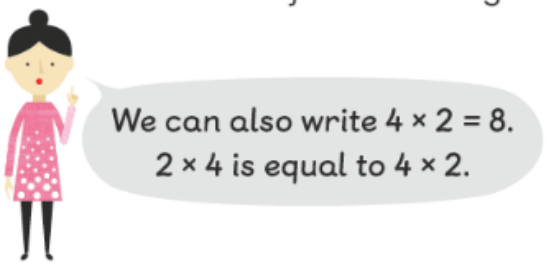


Number of groups

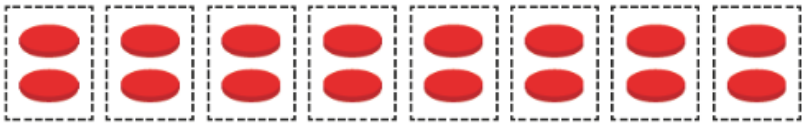
2 × 4 = 8

Number of objects in each group

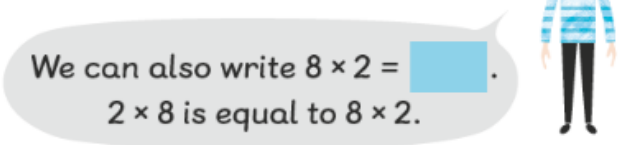
This is 4 groups of 2.



Multiply 2 by 8.



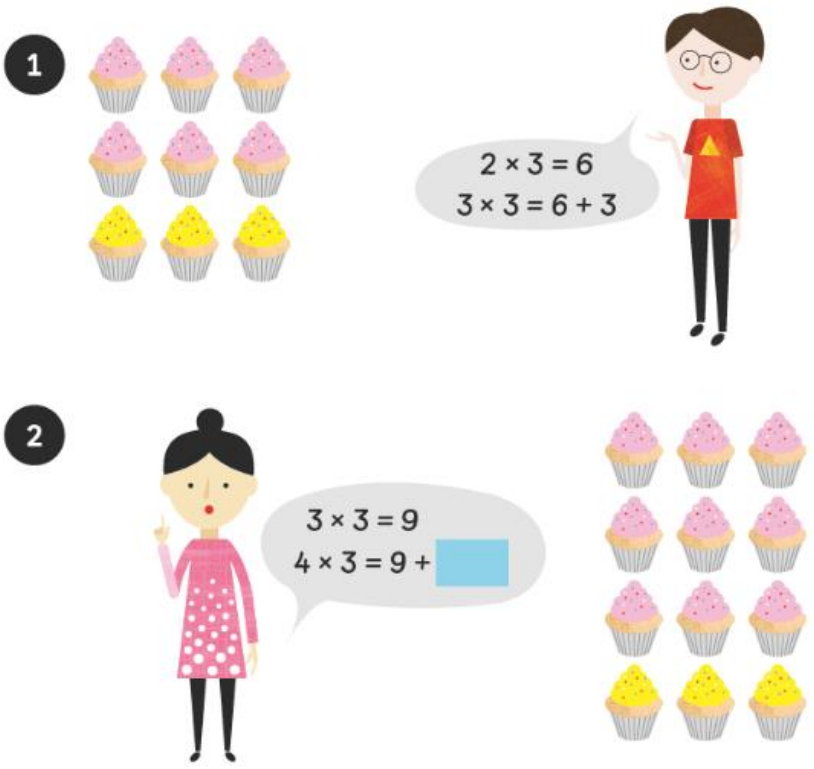
2 × 8 = ☐



Year 3

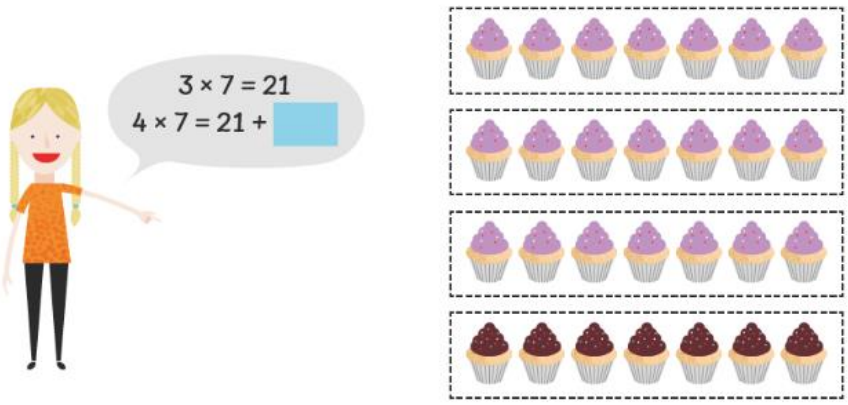
Multiplying by 3, 4 and 8

Number facts must be memorised and used on a daily basis. The school's KIRFs outline which facts are needed in each year group.



Year 4

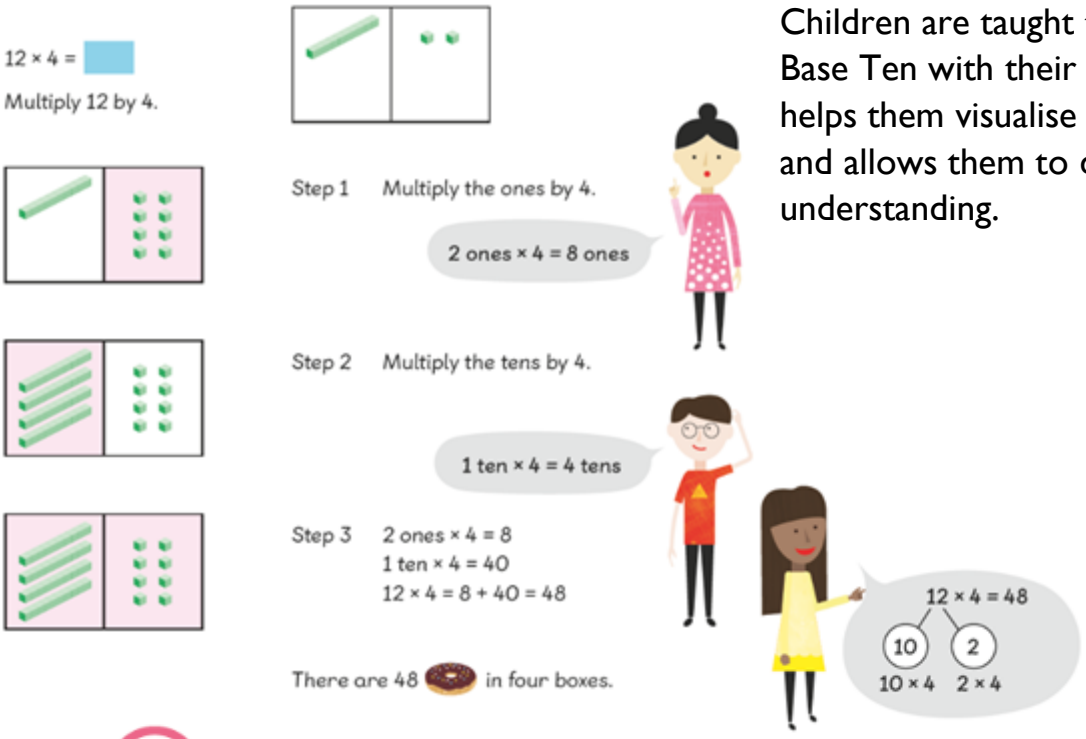
Multiplying by 6, 7, 9, 11 and 12



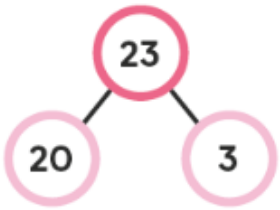
Formal Methods

Year 3

Multiplication with no Regrouping



Children are taught to use Dienes or Base Ten with their methods. This helps them visualise what is happening and allows them to develop a deeper understanding.



Step 1 Multiply the ones by 2.

3 ones × 2 = 6 ones

	t	o
	2	3
×		2
		6

Step 2 Multiply the tens by 2.

2 tens × 2 = 4 tens

	t	o
	2	3
×		2
		6
	4	0

Step 3 Add the products.

6 + 40 = 46

	t	o
	2	3
×		2
		6
+	4	0
	4	6

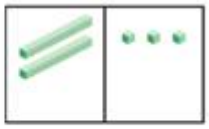
23 × 2 = 46

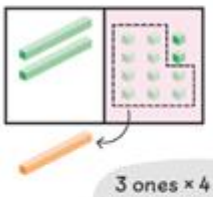
There are 46 children in the 2 classes.

Year 3

Multiplication with Regrouping

There are 4 groups of 23 fish.
How do we multiply 23 by 4?

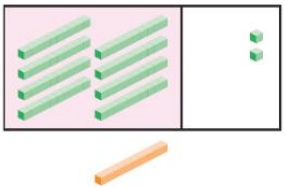




Step 1 Multiply the ones by 4.

	t	o
	2	3
×		4
	1	2

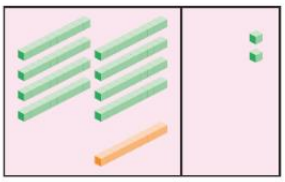
3 ones × 4 = 12 ones
12 ones = 1 ten 2 ones



Step 2 Multiply the tens by 4.

	t	o
	2	3
×		4
	1	2
	8	0

2 tens × 4 = 8 tens



Step 3 Add the products.

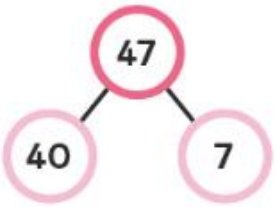
	t	o
	2	3
×		4
	1	2
+	8	0
	9	2

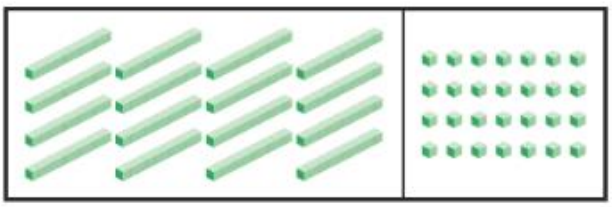
12 + 80 = 92

23 × 4 = 92

This is an expanded method showing the answer for multiplying 3x4 first and 20x4 second.

There are 92 fish in 4 tanks.
This is 47.





Step 1 Multiply the ones by 4.

7 ones × 4 = 28 ones
28 ones = 2 tens + 8 ones

2 tens

	t	o
	2	
	4	7
×		4
		8

8 ones

Step 2 Multiply the tens by 4.

4 tens × 4 = 16 tens
16 tens + 2 tens = 18 tens

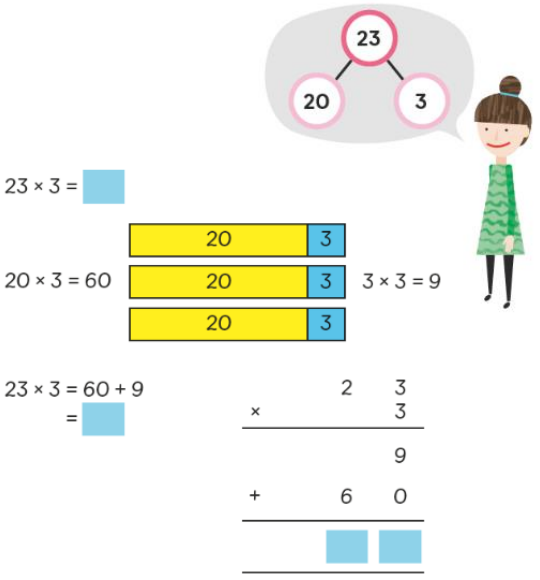
	h	t	o
		2	
		4	7
×			4
	1	8	8

47 × 4 = 188

Hannah is correct.

Formal Methods

Year 4



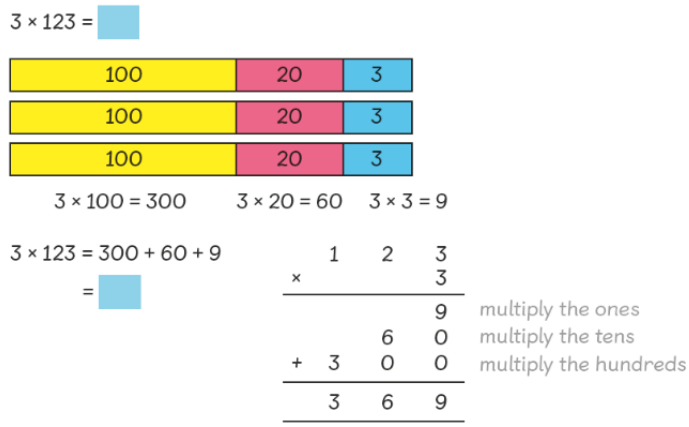
Children use pictorial representations to help them develop their understanding of multiplication. This reminds the children of the basic principles of multiplication as the numbers get more difficult.

For example, 23 x 3 is:

'3 lots of 23' or

'3 lots of 20 and 3 lots of 3' or

'3 x 20 + 3 x 3'

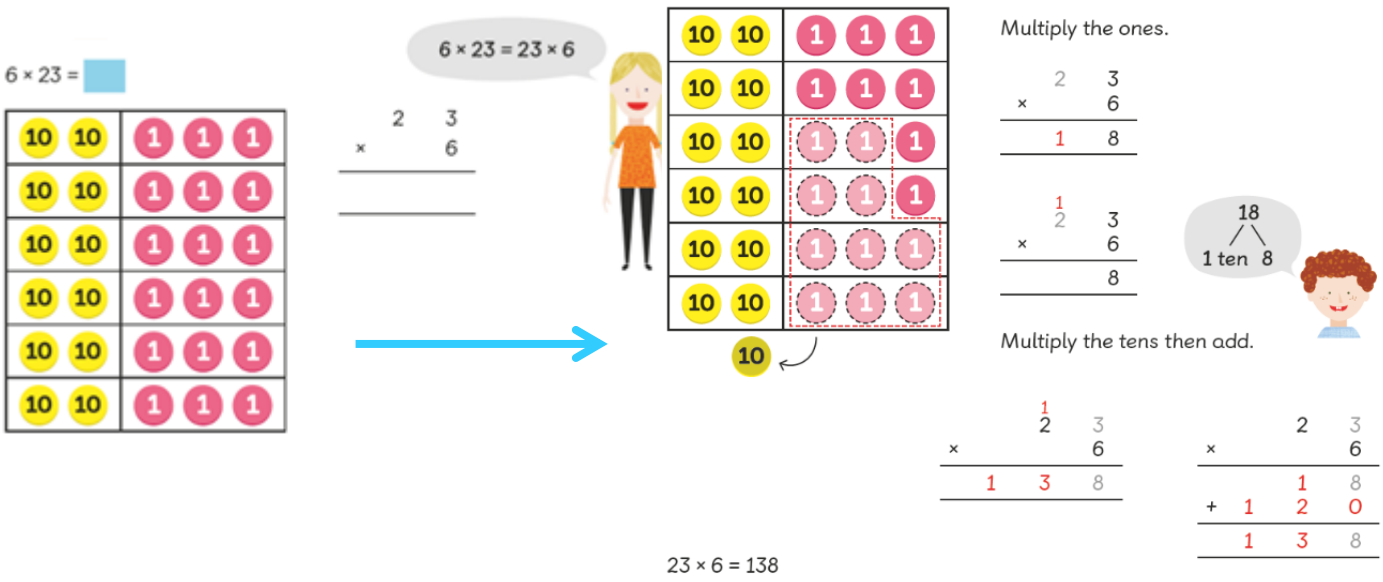


For example, 123 x 3 is:

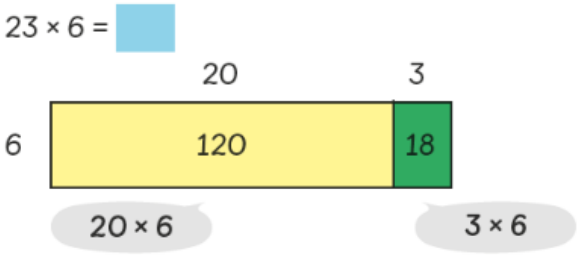
'3 lots of 123' or

'3 lots of 100 and 3 lots of 20 and 3 lots of 3' or

'3 x 100 + 3 x 20 + 3 x 3'



Year 4



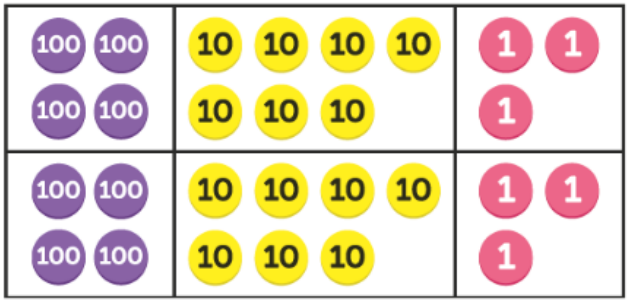
23 × 6 = 120 + 18
= 138

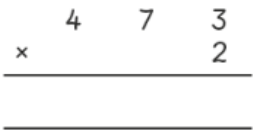
The children are taught pictorial representations that show what they have been doing with the equipment.

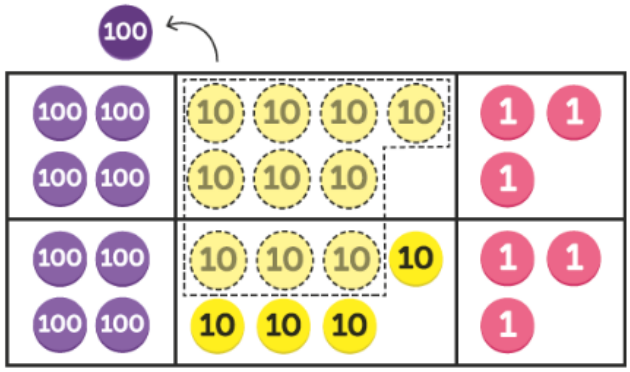
Here the counter have been replaced with a single block.

473 × 2 =

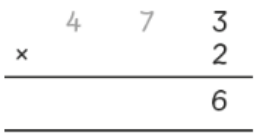
Children continue to use equipment as the numbers that they work with become larger.



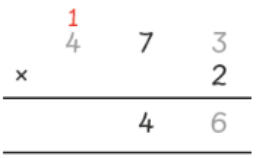




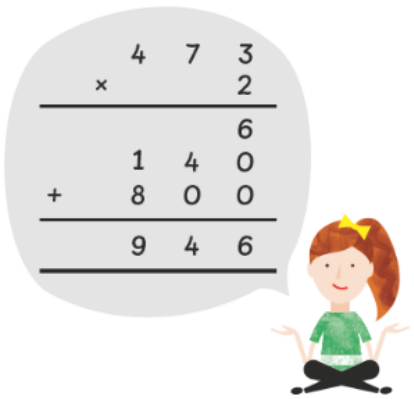
Multiply the ones.

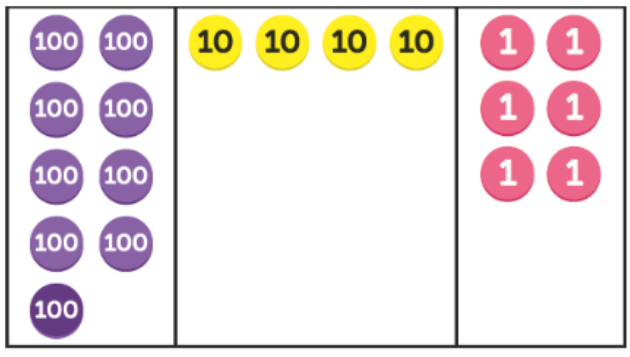


Multiply the tens.

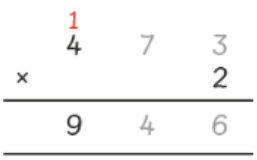


The compact method on the right replaces the expanded method below.





Multiply the hundreds.
Add the 1 hundred.



473 × 2 = 946