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*know, love, serve; God, Ourselves & Others*

# Mathematics

## Division Methods

This booklet covers the methods that we teach the children to use in school. While there are specific methods that we teach in each Year Group the children will have methods that they feel more confident with. Our aim is to develop an understanding of the Maths behind these methods rather than teaching the children steps in a process so talking and explaining these methods is as important as carrying them out.

Other maths material provided by the school includes:

- Creating Mathematicians
- Counting, Addition, Subtraction, Multiplication and Division Methods
- Half termly, Key Instant Recall Facts (KIRFs)
- Topic, Knowledge Organisers



PRIMARY MATHS SERIES

**SCHEME OF WORK – YEAR 4**



For more information on the what is covered in each year group our Schemes of Work are published on our website and include:

- An overview of the national curriculum topics covered during the school year by term
- A full lesson breakdown for each national curriculum topic and the learning objective for each lesson



## Division Methods

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## CPA Approach

Concrete, Pictorial, Abstract (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths in children. Often referred to as the concrete, representational, abstract



framework, CPA was developed by American psychologist Jerome Bruner. It is an essential technique within the Singapore method of teaching maths for mastery.

### **At a glance**

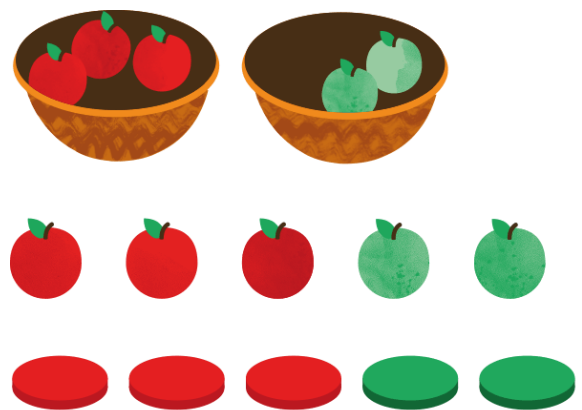
- An essential technique of maths mastery that builds on a child's existing understanding
- A highly effective framework for progressing pupils to abstract concepts like fractions
- Involves concrete materials and pictorial/representational diagrams
- Based on research by psychologist Jerome Bruner
- Along with [bar modelling](#) and [number bonds](#), it is an essential maths mastery strategy

### **Background to the CPA framework**

Children (and adults!) can find maths difficult because it is abstract. The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems. The CPA framework is so established in Singapore maths teaching that the Ministry of Education will not approve any teaching materials that do not use the approach.

### **Concrete step of CPA**

Concrete is the “doing” stage. During this stage, students use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.



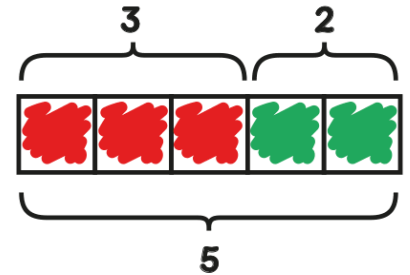
For example, if a problem involves adding pieces of fruit, children can first handle actual fruit. From there, they can progress to handling abstract counters or cubes which represent the fruit.

## Pictorial step of CPA



Pictorial is the “seeing” stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps students visualise abstract problems and make them more accessible.



## Abstract step of CPA

Abstract is the “symbolic” stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, −, ×, /) to indicate addition, multiplication or division.

$$3 + 2 = \boxed{5}$$

## Making CPA work

Although we’ve presented CPA as three distinct stages, a skilled teacher will go back and forth between each stage to reinforce concepts.

The [MNP Primary Series](#) approach encourages teachers to vary the apparatus that children use in class. For example, students might one day use counters, another day they might use a ten frame. Likewise, children are encouraged to represent the day’s maths problem in a variety of ways. For example, drawing an array, a number bond diagram or a bar model.

By systematically varying the apparatus and methods used to solve a problem, children can craft powerful mental connections between the concrete, pictorial, and abstract phases.

When teaching young children numbers, counters and multi-link cubes are more commonly used. However, removing concrete materials exposes children to abstract concepts too early. As a result, they miss out on the opportunity to build a conceptual mathematical understanding that can propel them through their education.

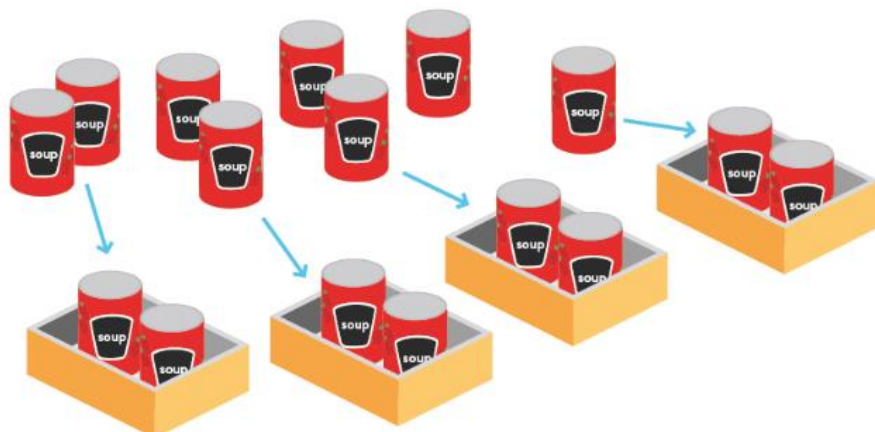
It is important to recognise that the CPA model is a progression. By the end of KSI, children need to be able to go beyond the use of concrete equipment to access learning using either pictorial representations or abstract understanding. What is important, therefore, is that all learners, however young, can see the connections between each representation.

# Grouping Equally

## Year 1

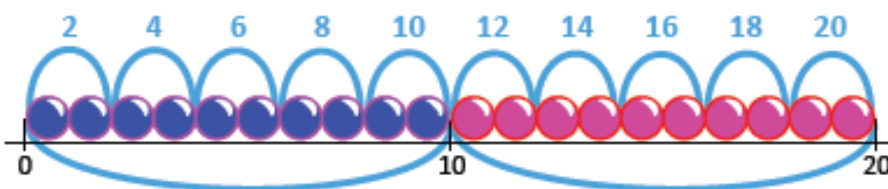
In Year 1 children begin to use visual and concrete arrays and 'sets of objects' to find answers to questions.

There are 8 cans.



There are 4 boxes of 2 cans.

'Clever' counting in 2s and 10s is practised and pictorial representations are shown to the children.



## Year 2

There are 20 chocolates.



Put 2 chocolates in each bag.



Emma gets 10 bags of chocolate.

$20 \div 2 = 10$  is a division equation.

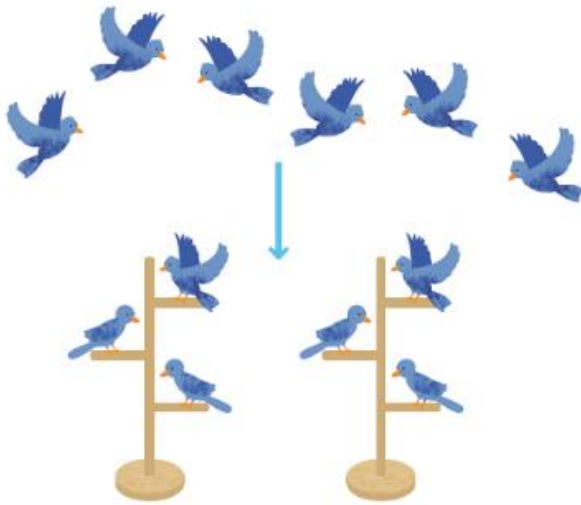
$20 \div 2 = 10$  is read as twenty divided by two equals ten.

Relating division to multiplication facts ( $10 \times 2 = 20$ ) will improve the speed at which the children carry out operations and it is important for them to practise saying division statements as well as multiplication facts.

# Grouping Equally

## Year 3

Put the birds in groups of 3.



Divide 6 by 3 to find the number of stands needed.

$$2 \times 3 = 6$$
$$6 \div 3 = 2$$

$$6 \div 3 = 2$$

At least 2 stands are needed.

In Year 3, children will also need to recognise that division is not commutative eg  $16 \div 8$  does not equal  $8 \div 16$

They will continue to practise saying statements in different ways.



## Year 4



A roller coaster with 6 carriages, all of the same size, can seat 24 people.  
How many people does each carriage seat?



$$24 \div 6 = 4$$

$$6 \times 4 = 24$$
$$24 \div 6 = 4$$

Each carriage can seat 4 people.  
When 24 is divided by 6, the quotient is 4.



# Sharing Equally

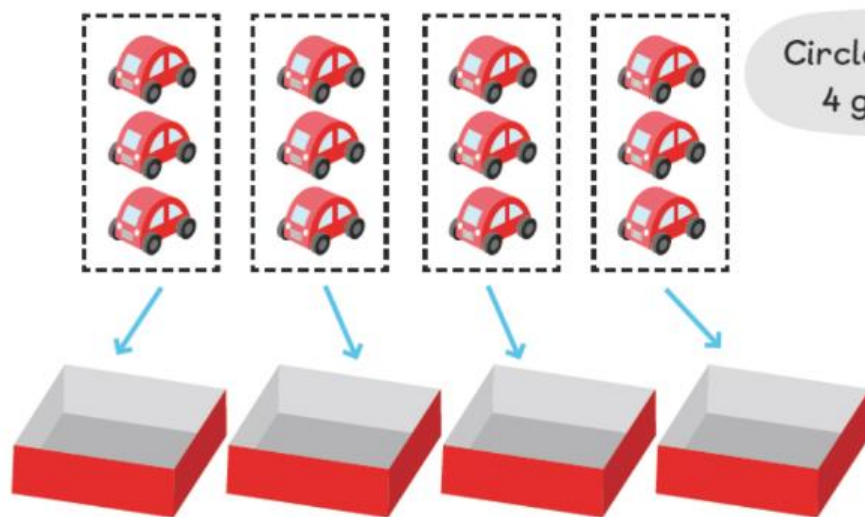
## Year 1

Children will find half of even numbers up to 12 including realising that it is hard to halve an odd number.

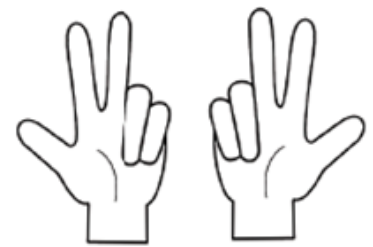
There are 12 toy cars.

Put the toy cars equally into 4 boxes.

How many toy cars are there in each box?



Circle to make 4 groups.

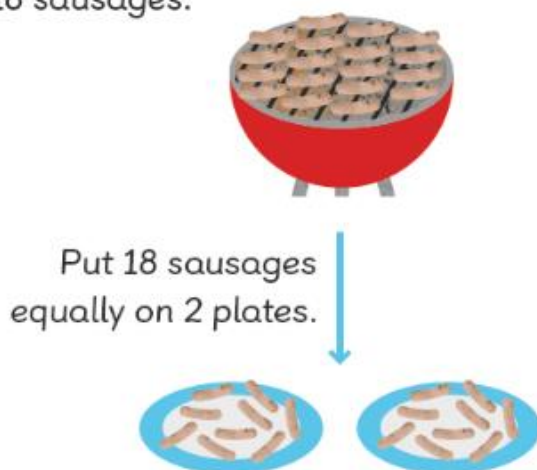


Children will share by putting one in each box repeatedly. Objects will be on hand for the children to use as well as counters or blocks that can represent the objects.

There are 3 toy cars in each box.

## Year 2

There are 18 sausages.



Put 18 sausages equally on 2 plates.

$$2 \times 9 = 18$$




There are 9 sausages on each plate.


$$18 \div 2 = 9$$

Links between division facts and multiplication facts continue to be made as more tables are learnt.

## Year 3

Children learn to divide by 3 and 4.

Divide 12 by 4 to find the number of  in each box.

$12 \div 4 = 3$   
There are 3  in each box.

$12 \div 4 = 3$   
 $3 \times 4 = 12$

## Year 4

Children learn to divide by 6, 7 and 9

1

Divide 21 by 7 to find the number of cupcakes in each box.

$21 \div 7 = 3$

$21 \div 7 = 3$   
 $7 \times 3 = 21$

## Dividing with Remainders

There were 11 balloons.

There are 3 cupcakes in each box.

$11 \div 2 = 5 \text{ remainder } 1$

The quotient is 5 and the remainder is 1.  
Each friend got 5 balloons.  
There was 1 balloon left over.

# Using known Number Facts

## Year 3

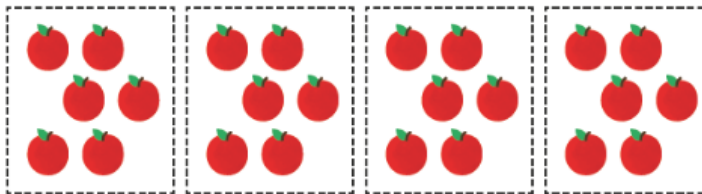
Number facts must be memorised and used on a daily basis.

Children should know halves of multiples of 10 to 200, e.g. half of 170 is 85.

Know 2x, 3x, 4x, 5x, 8x, 10x division facts.

Use division facts to find simple fractions of amounts within the times tables.

- 1 Put 24 apples into 4 equal groups.

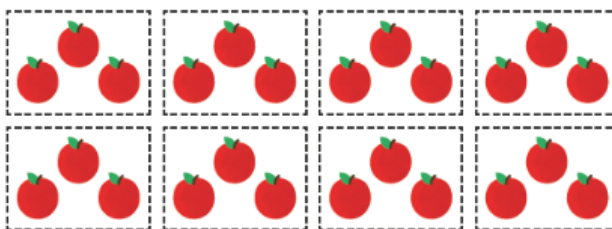


$24 \div 4 = 6$   
Each group has 6 apples.

$4 \times 6 = 24$



- 2 Put 24 apples into 8 equal groups.



$24 \div 8 = 3$   
Each group has 3 apples.

$8 \times 3 = 24$



## Year 4

Children will use their knowledge of times table facts, up to 12x12, and all related division facts.

Making 2 groups of 12



24



12 beans multiplied  
2 times

$2 \times 12 = 24$

2 groups



We can also write  $12 \times 2 = 24$ .

Putting 24 beans into 2 equal groups

$24 \div 2 = 12$

Each group has  
12 beans.



Putting 24 beans into groups of 12

$24 \div 12 = 2$

There are 2 groups.



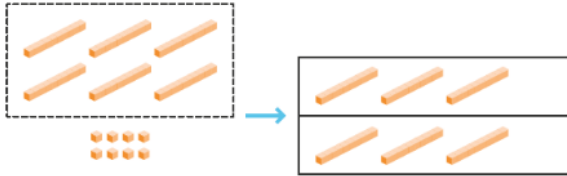
## Year 3

### Diving without Regrouping

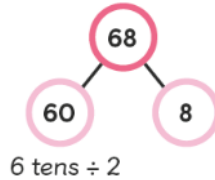
To find the number of sweets each person gets, divide 68 by 2.

$$68 \div 2 = \square$$

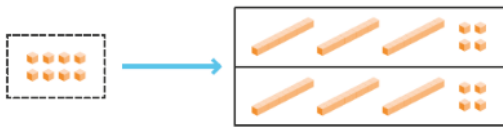
Step 1 Divide 6 tens by 2.



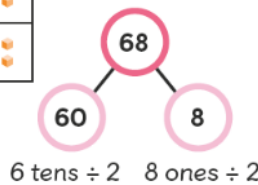
6 tens  $\div$  2  
= 3 tens



Step 2 Divide 8 ones by 2.



8 ones  $\div$  2  
= 4 ones

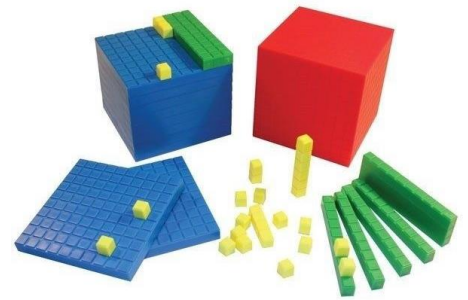


Step 3 Add the results.

$$68 \div 2 = 30 + 4 = 34$$

Each person gets 34 sweets.

Children use Dienes or Base Ten Blocks to develop their understanding of larger numbers and the place value of each digit.

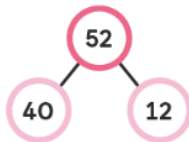
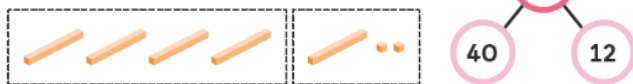


### Dividing with Regrouping

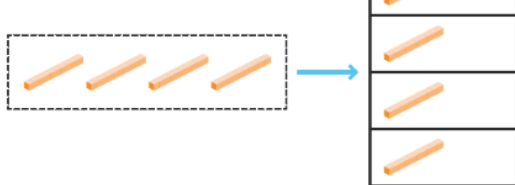
To find the number of ice creams in each box, divide 52 by 4.

$$52 \div 4 = \square$$

Step 1 Split 52 into 40 and 12.



Step 2 Divide the tens by 4.



4 tens  $\div$  4 = 1 ten



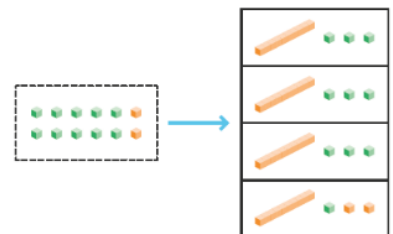
Step 3 Regroup 1 ten into 10 ones.



12 ones  $\div$  4 = 3 ones



Step 4 Divide the ones by 4.



Step 5 Add the results.

$$52 \div 4 = 10 + 3 = 13$$

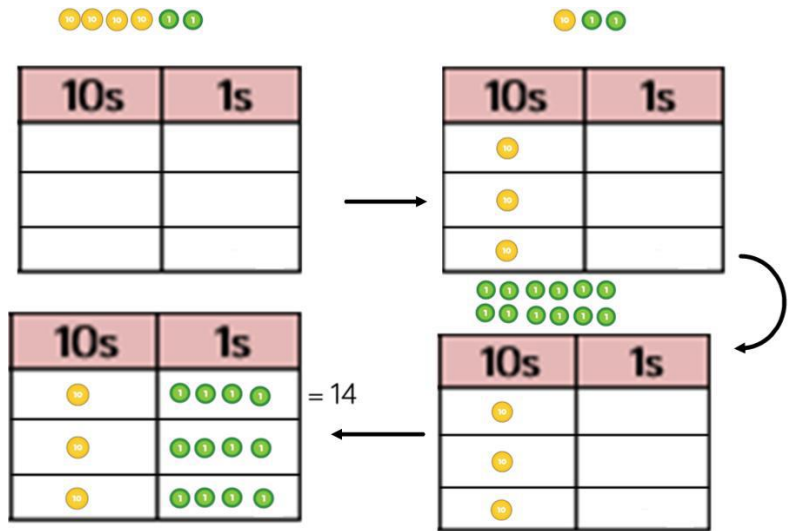
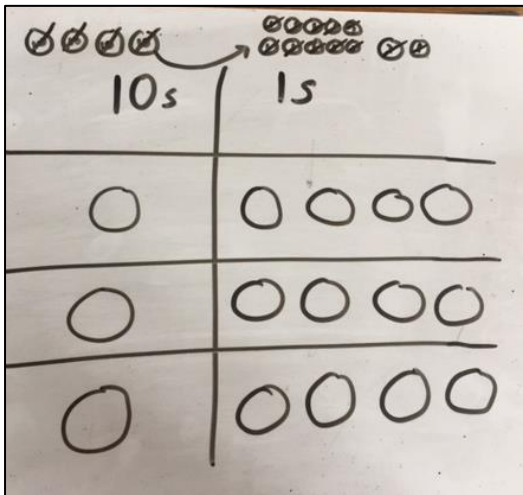
There are 13 ice creams in each box.

# Informal Methods

## Year 4

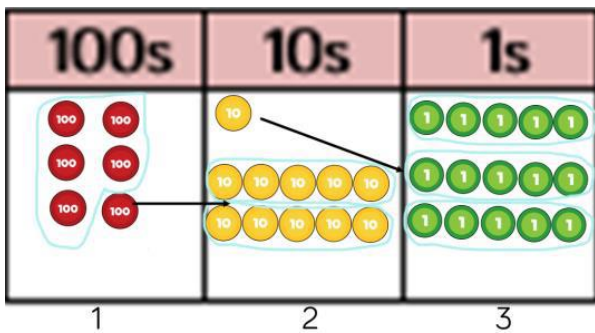
Sharing using place value counters  $42 \div 3 = 14$

3 groups to share into as  $\div 3$

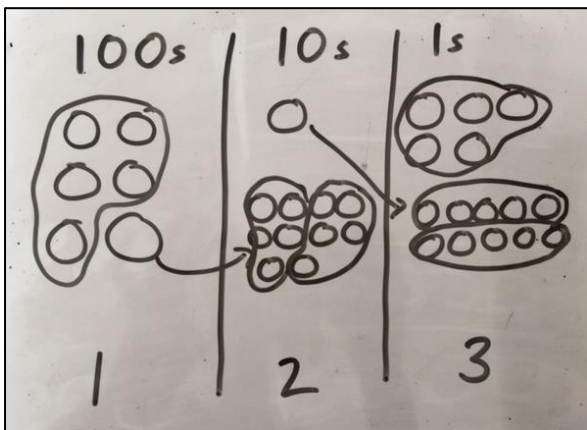


Children represent place value counters pictorially.

Short division using place value counters to group.  $615 \div 5$

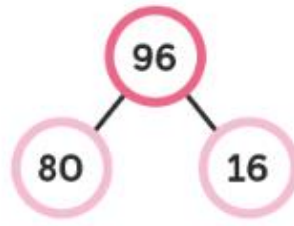


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?



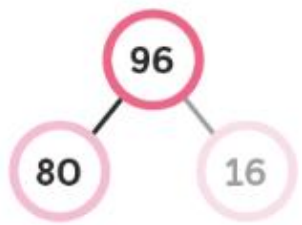
Represent the place value counters pictorially.

## Year 3



First, I take 80 from 96.  
Then, I take 16 from the remaining 16.

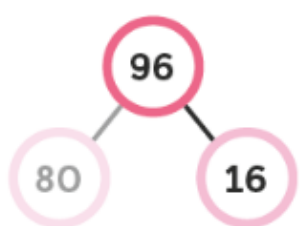
$$\begin{array}{r}
 12 \\
 8 \overline{) 96} \\
 \underline{- 80} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$



1 ten

$$\begin{array}{r}
 12 \\
 8 \overline{) 96} \\
 \underline{- 80} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$

$8 \text{ tens} \div 8 = 1 \text{ ten}$



2 ones

$$\begin{array}{r}
 12 \\
 8 \overline{) 96} \\
 \underline{- 80} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$

$16 \text{ ones} \div 8 = 2 \text{ ones}$

$1 \text{ ten} + 2 \text{ ones} = 12$

$$\begin{array}{r}
 12 \\
 8 \overline{) 96} \\
 \underline{- 80} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$

$96 \div 8 = 12$

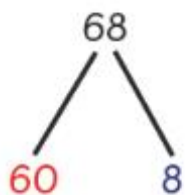
# Formal Methods

## Year 4

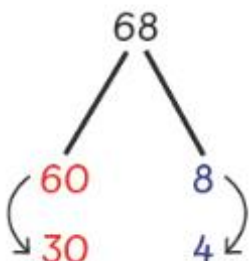
$68 \div 2 = \square$



### Method 1



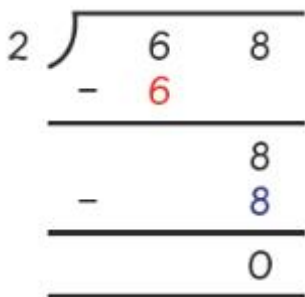
$68 = 60 + 8$



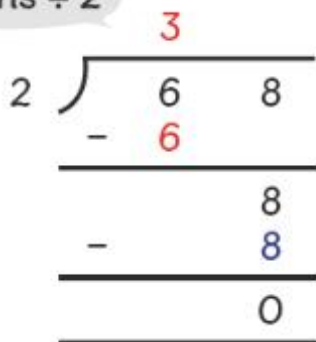
Divide 60. Divide 8.



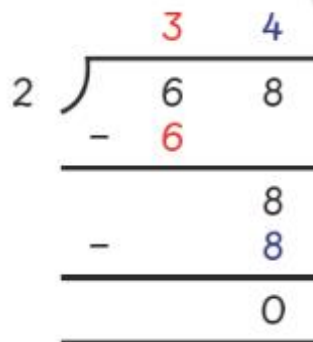
### Method 2



6 tens  $\div$  2



8 ones  $\div$  2

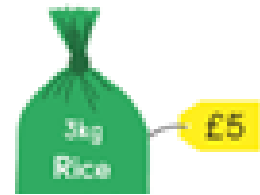


Each should take 34 postcards.

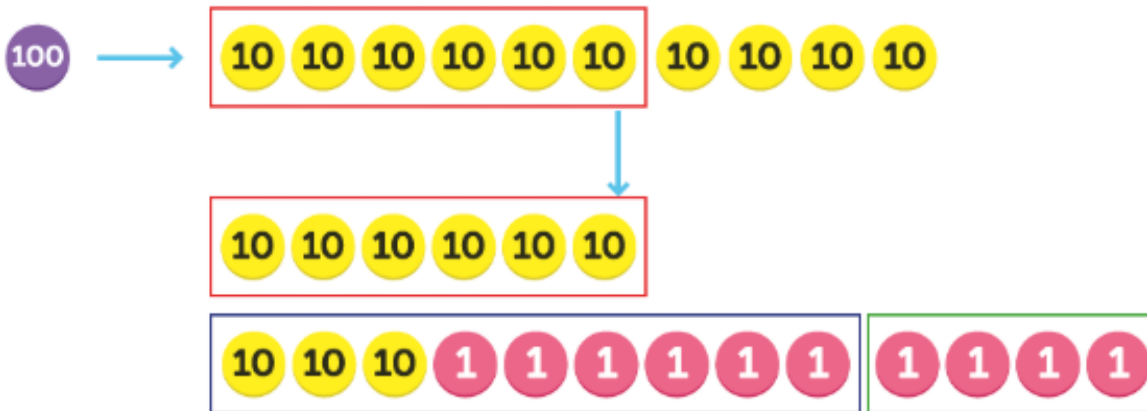
$68 \div 2 = 34$

## Year 4

A shopkeeper repacks 100 kg of rice into 3-kg bags to sell.  
How many bags does he get?

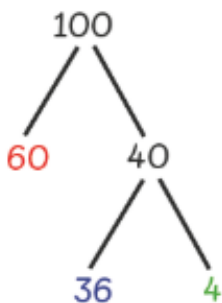


What if he wants to pack the rice into 6-kg bags?

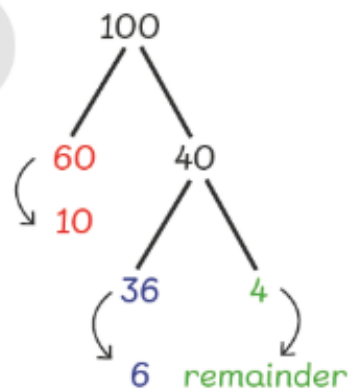


Take 60 from 100. 40 is left.  
Take 36 from 40. 4 is left.

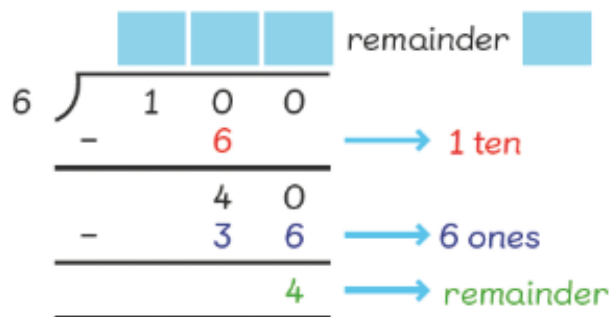
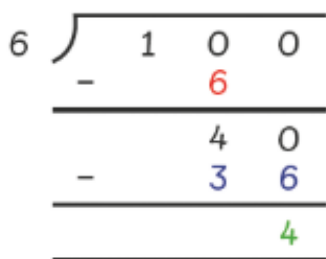
Method 1



Divide 60. Divide 36.  
We can get a  
quotient.



Method 2



$$100 \div 6 = 16 \text{ remainder } 4$$

He gets 16 bags and a remainder of 4 kg of rice.