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know, love, serve; God, Ourselves & Others

Mathematics
Addition Methods

This booklet covers the methods that we teach the children to use in school. While there are specific methods that we teach in each Year Group the children will have methods that they feel more confident with. Our aim is to develop an understanding of the Maths behind these methods rather than teaching the children steps in a process so talking and explaining these methods is as important as carrying them out.

Other maths material provided by the school includes:

- Creating Mathematicians
- Counting, Addition, Subtraction, Multiplication and Division Methods
- Half termly, Key Instant Recall Facts (KIRFs)
- Topic, Knowledge Organisers

MATHS NO PROBLEM!

PRIMARY MATHS SERIES
SCHEME OF WORK – YEAR 4

For more information on the what is covered in each year group our Schemes of Work are published on our website and include:

- An overview of the national curriculum topics covered during the school year by term
- A full lesson breakdown for each national curriculum topic and the learning objective for each lesson

CPA Approach

Concrete, Pictorial, Abstract (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths in children. Often referred to as the concrete, representational, abstract framework, CPA was developed by American psychologist Jerome Bruner. It is an essential technique within the Singapore method of teaching maths for mastery.

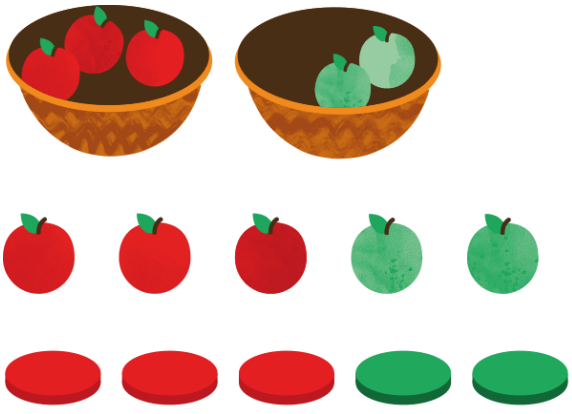
At a glance

- An essential technique of maths mastery that builds on a child's existing understanding
- A highly effective framework for progressing pupils to abstract concepts like fractions
- Involves concrete materials and pictorial/representational diagrams
- Based on research by psychologist Jerome Bruner
- Along with bar modelling and number bonds, it is an essential maths mastery strategy

Background to the CPA framework

Children (and adults!) can find maths difficult because it is abstract. The CPA approach builds on children's existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems. The CPA framework is so established in Singapore maths teaching that the Ministry of Education will not approve any teaching materials that do not use the approach.

Concrete step of CPA



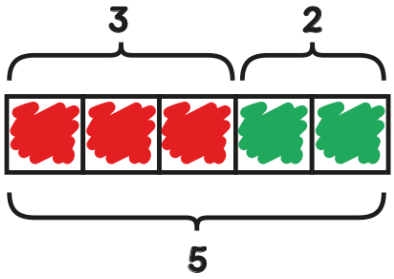
Concrete is the "doing" stage. During this stage, students use concrete objects to model problems. Unlike traditional maths teaching methods where teachers demonstrate how to solve a problem, the CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.

For example, if a problem involves adding pieces of fruit, children can first handle actual fruit. From there, they can progress to handling abstract counters or cubes which represent the fruit.

Pictorial step of CPA

Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Simply put, it helps students visualise abstract problems and make them more accessible.



Abstract step of CPA

Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, –, x, /) to indicate addition, multiplication or division.

$$3 + 2 = \boxed{5}$$

Making CPA work

Although we've presented CPA as three distinct stages, a skilled teacher will go back and forth between each stage to reinforce concepts.

The MNP Primary Series approach encourages teachers to vary the apparatus that children use in class. For example, students might one day use counters, another day they might use a ten frame. Likewise, children are encouraged to represent the day's maths problem in a variety of ways. For example, drawing an array, a number bond diagram or a bar model.

By systematically varying the apparatus and methods used to solve a problem, children can craft powerful mental connections between the concrete, pictorial, and abstract phases.

When teaching young children numbers, counters and multi-link cubes are more commonly used. However, removing concrete materials exposes children to abstract concepts too early. As a result, they miss out on the opportunity to build a conceptual mathematical understanding that can propel them through their education.

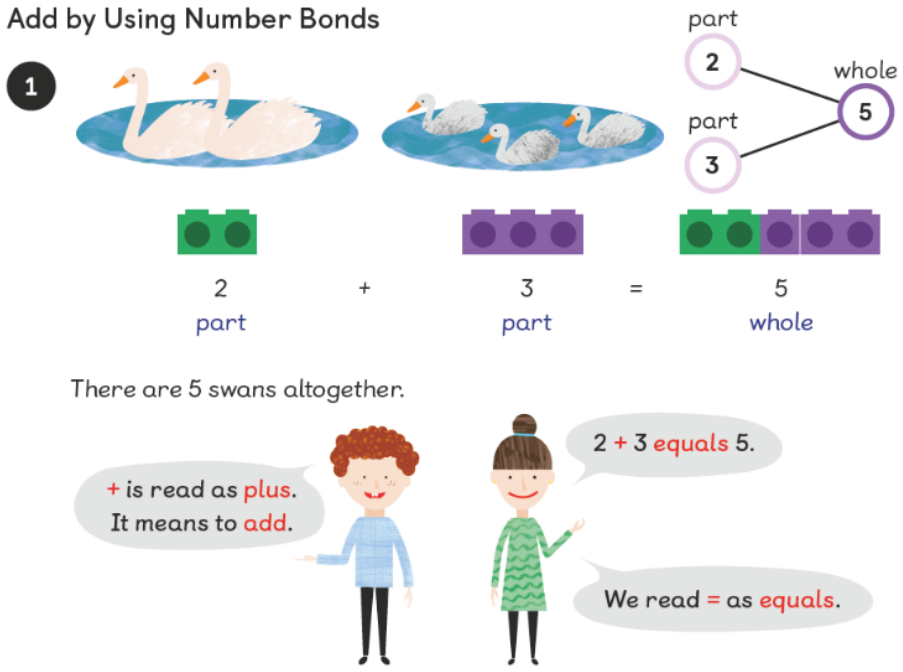
It is important to recognise that the CPA model is a progression. By the end of KS1, children need to be able to go beyond the use of concrete equipment to access learning using either pictorial representations or abstract understanding. What is important, therefore, is that all learners, however young, can see the connections between each representation.

Contents

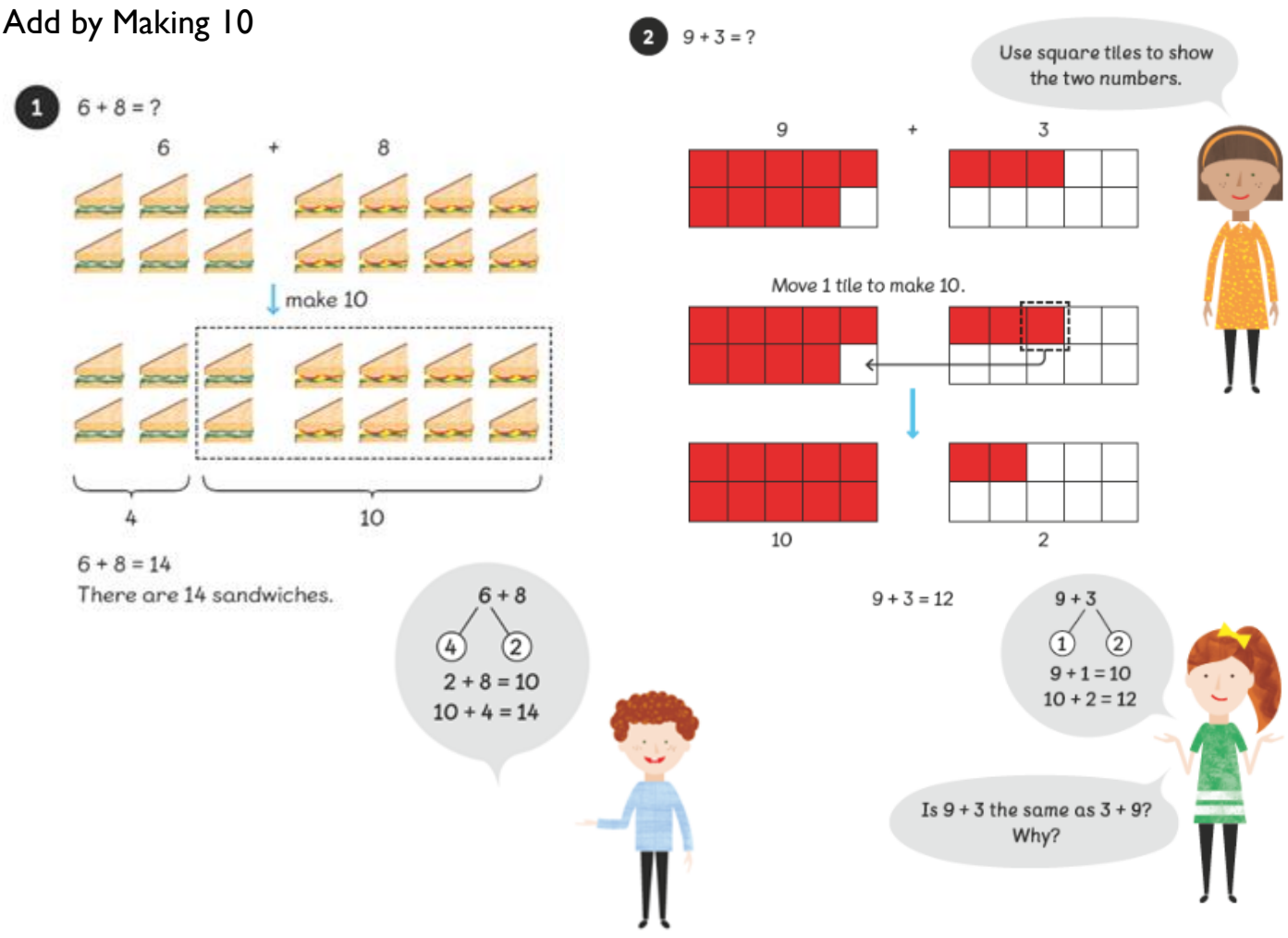
Addition Methods

Adding using Number Bonds & Number Facts

Year 1



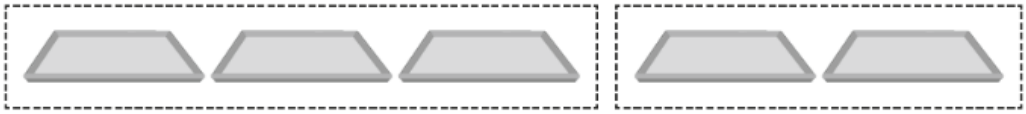
Children learn that a number is made up of other numbers. They find as many ways as possible to construct numbers. This is fundamental to all Mathematics as later methods taught require the children to be able to regroup numbers (subtraction) and rename (addition). This also helps the children develop quick mental strategies by making numbers that are easy to use (10s, 100s, etc)



Adding using Number Bonds & Number Facts

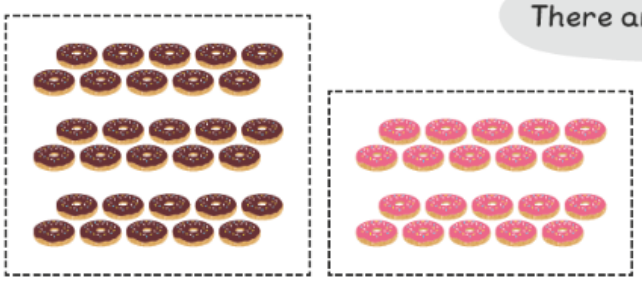
Year 2

Children continue to learn how basic number facts can help the with larger calculations for example making the numbers 10 times bigger.



Each tray holds 10 doughnuts

3 + 2 = 5



30 + 20 = 50

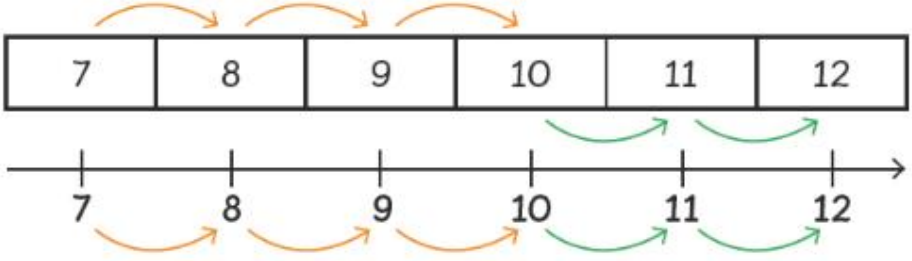
Recognising patterns in number is taught throughout the Curriculum it makes a Mathematician. Pairs of numbers that make 10 and that are doubles speed up addition calculations.

Add 7, 3 and 2.

Method 1 Make 10.

7 + 3 + 2 = 10 + 2
= 12

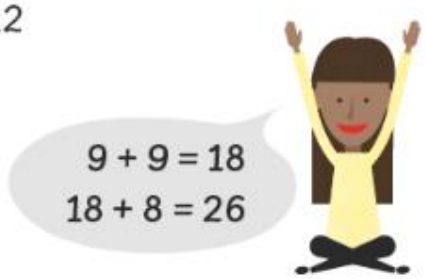
Method 2 Add by counting on.



7 + 3 + 2 = 12

Add 9, 9 and 8.

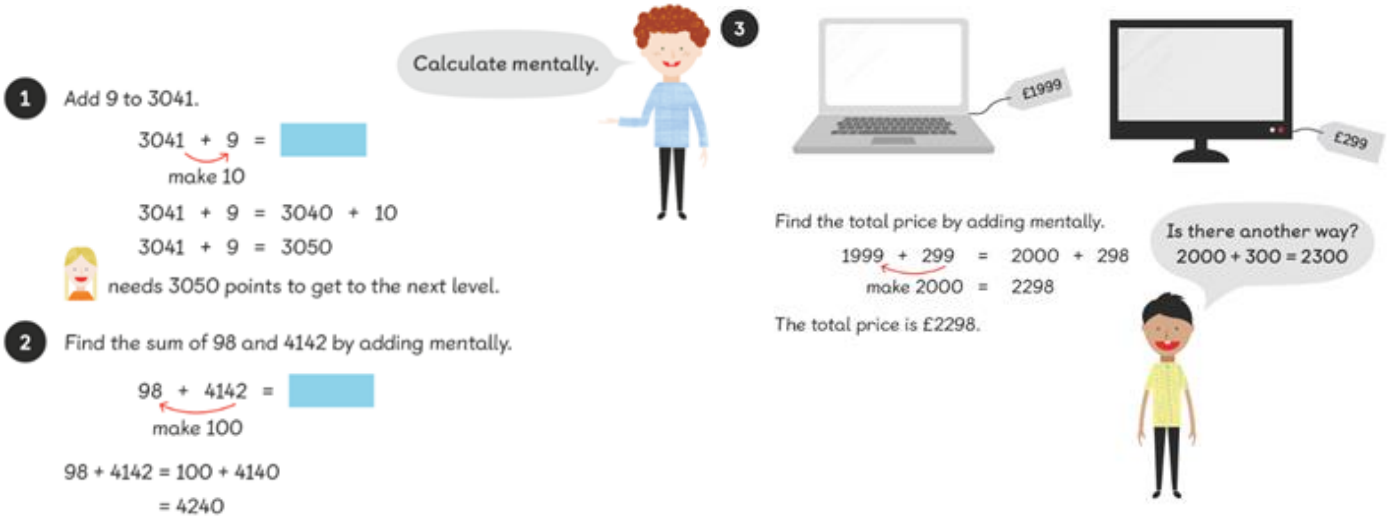
9 + 9 + 8 = 26



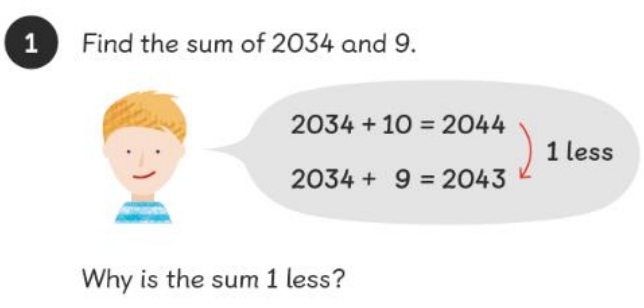
Adding using Number Bonds & Number Facts

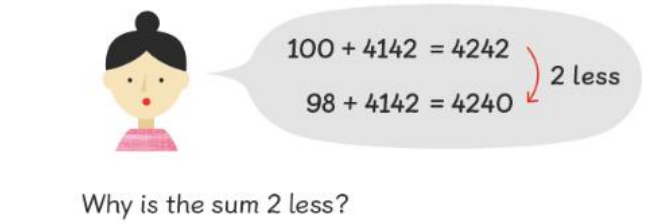
Year 3

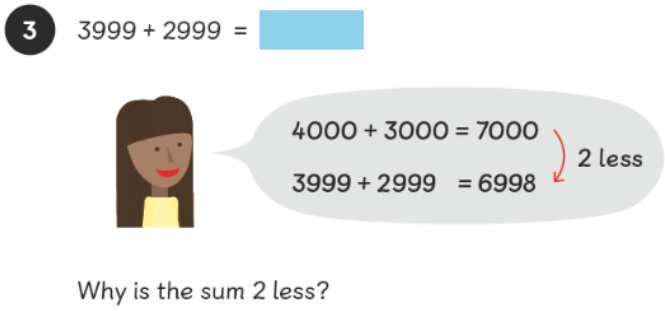
Year 4



Children use strategies and patterns learnt in previous years with increasingly large numbers. Number bonds to whole tens, hundreds and thousands are increasingly important.







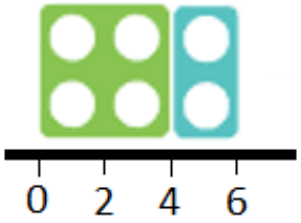
As children develop confidence with number bonds, they will start to use numbers close to these and adjusting. For example, +9 is the same as adding 10 but then taking 1 away (adjustment). They will make tens, hundreds and thousands.

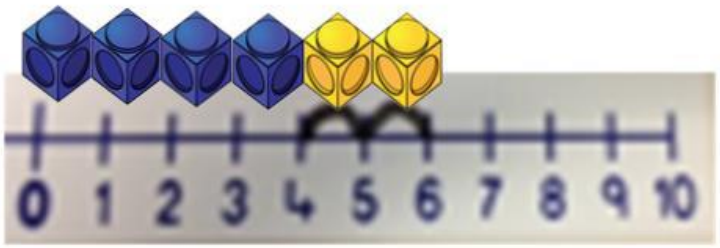
Adding by Counting On

Year 1

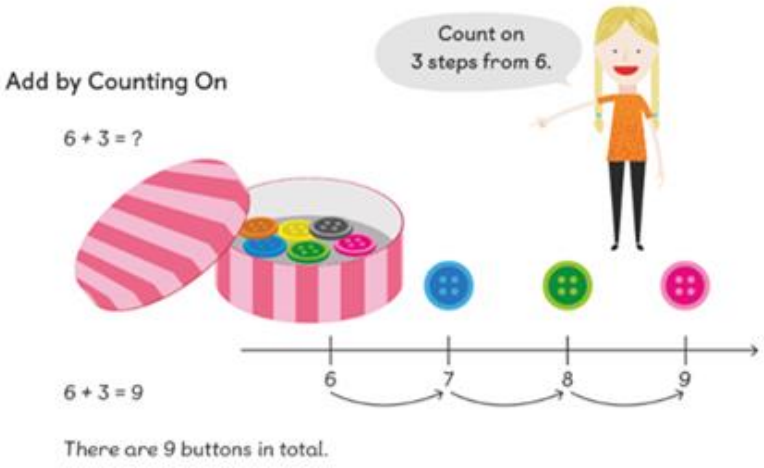
Children are taught to count on using a wide range of equipment. Initially we look at the how this looks when represented on a number line starting at 0 but it is important that the children develop an understanding that they do not need to recount their first number to find the total.

Numicon

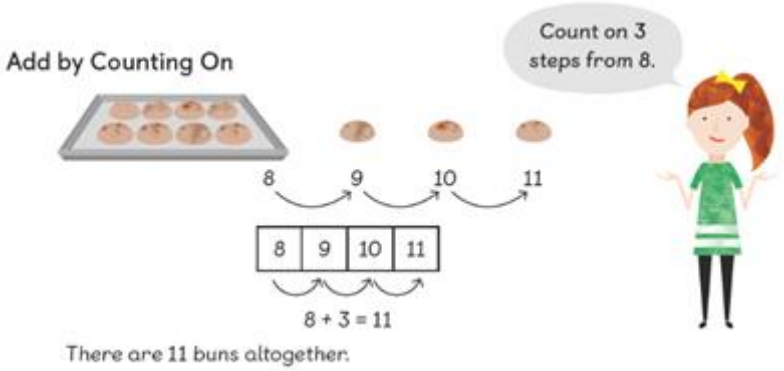




Multilink cubes



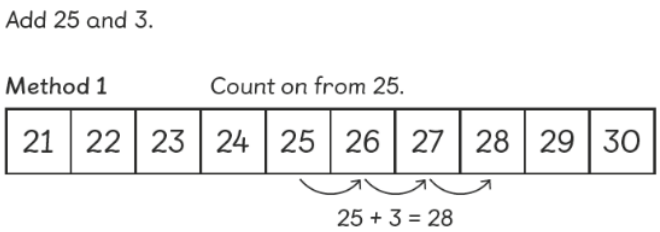
Here the initial number (including the objects if using concrete apparatus) is placed into a box to establish the first number doesn't need to be counted each time.

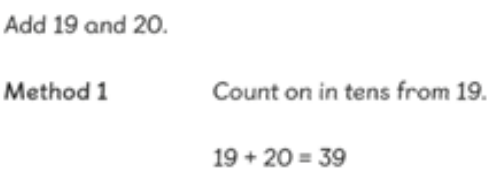


Counting on can be done on a number line (as in the button example) or on a number track (left).

Year 2

Children continue to practise counting on but will refine the method with jumps of different sizes to make their calculations more efficient. For example, being able to make a jump of 10 will help the children greatly.



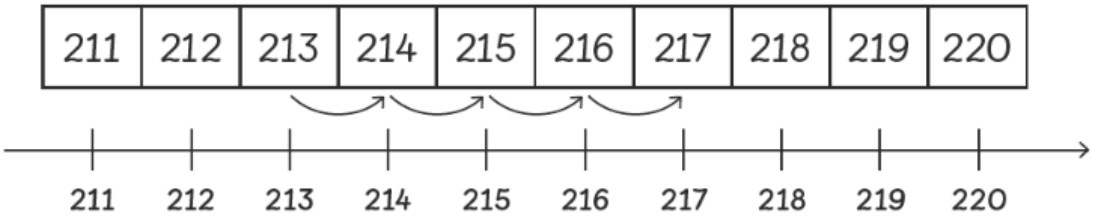


Year 3

Counting on Ones

Add 213 and 4.

Method 1 Count on from 213.



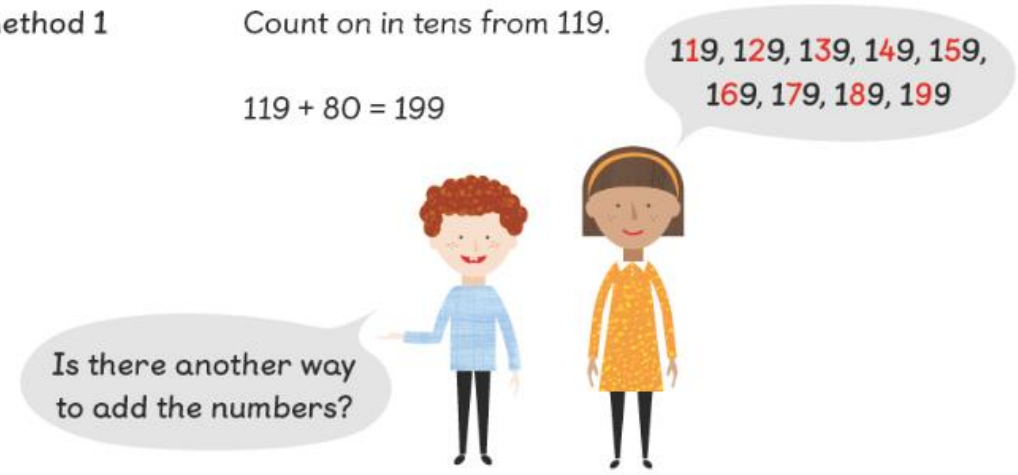
213 + 4 = 217

Counting on Tens

Add 119 and 80.

Method 1 Count on in tens from 119.

119 + 80 = 199

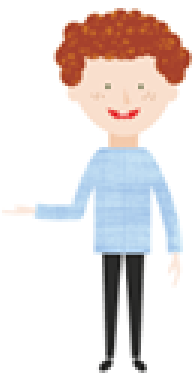


Counting on Hundreds

Add 213 and 400.

Method 1 Count on in hundreds from 213.

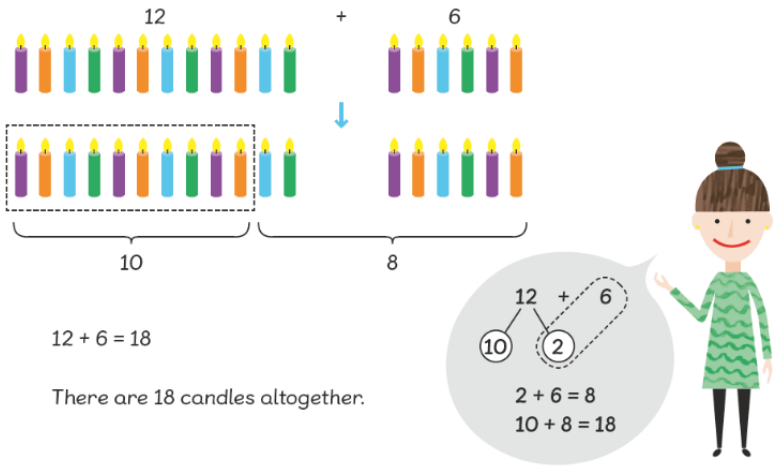
213 + 400 = 613



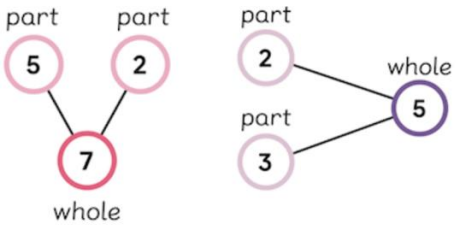
Adding Ones, Tens, Hundreds, …

Year 1

Adding Ones



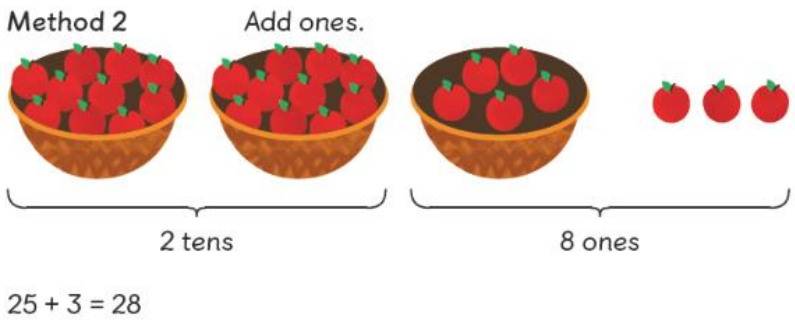
Children use the Part Whole Model:



To help them organise their thinking and explain what they are adding.

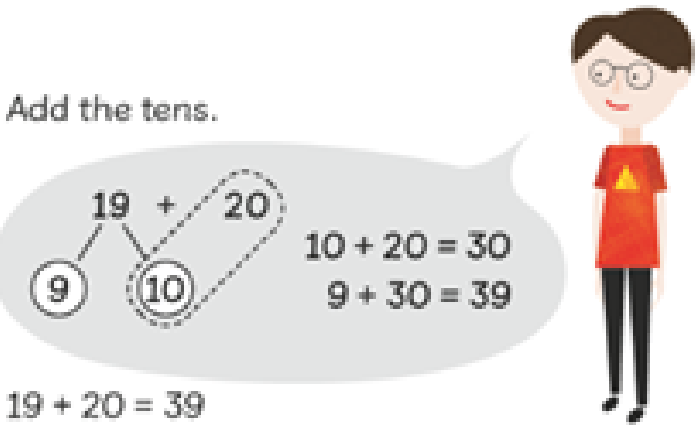
Year 2

Adding Ones



Children are encouraged to talk about their methods and show using concrete objects.

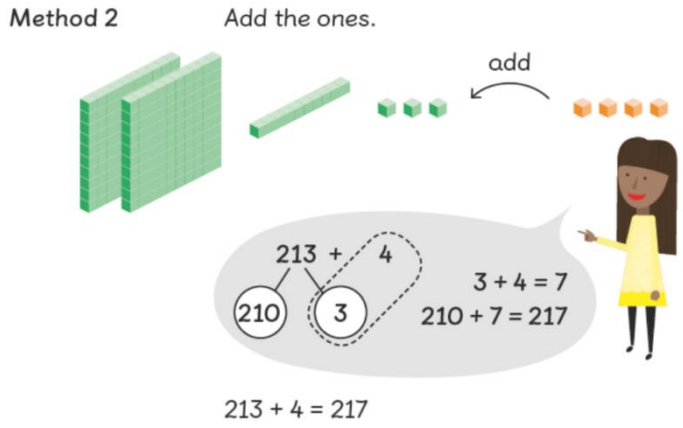
Adding Tens



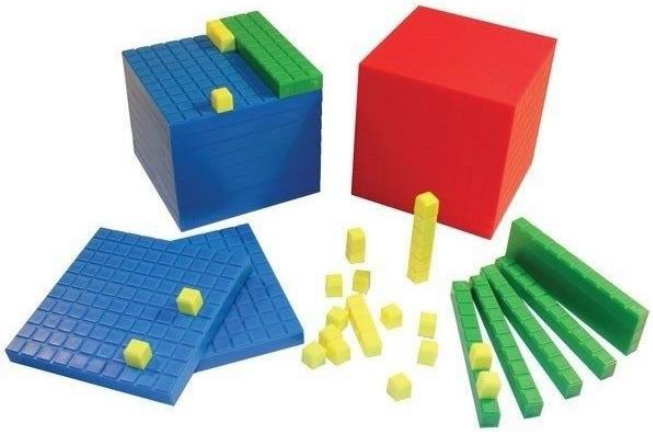
Children extend their understanding of the addition of tens by recognising its relationship to adding ones.

Year 3

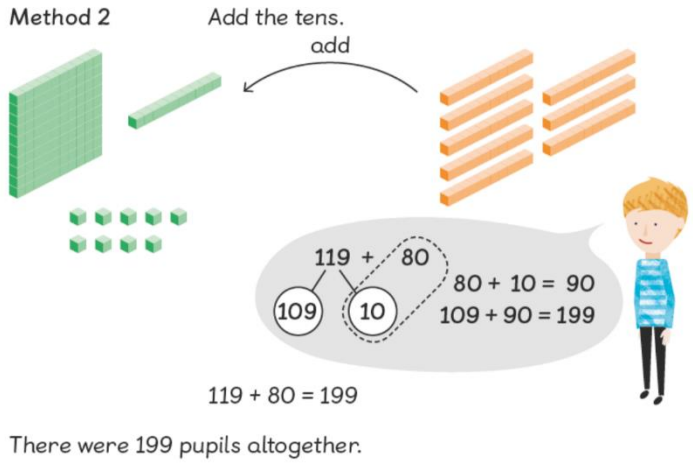
Adding Ones



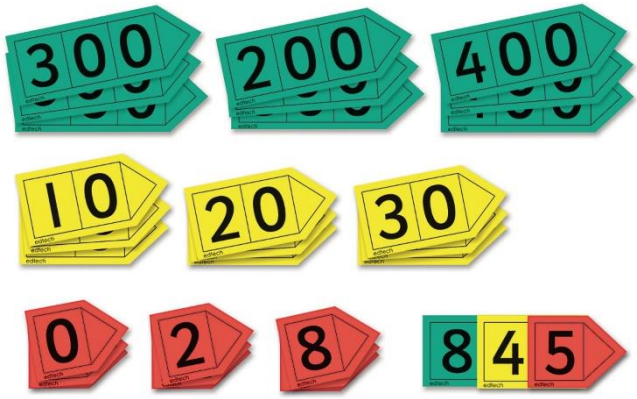
Children use Dienes or Base Ten Blocks to develop their understanding of larger numbers and the place value of each digit.



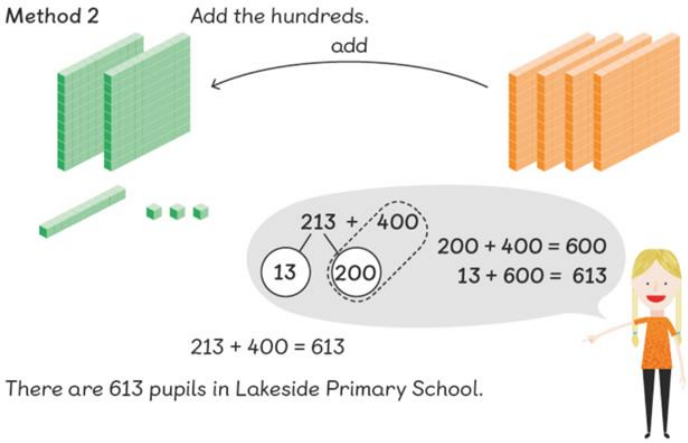
Adding Tens



Place value cards also help.



Adding Hundreds

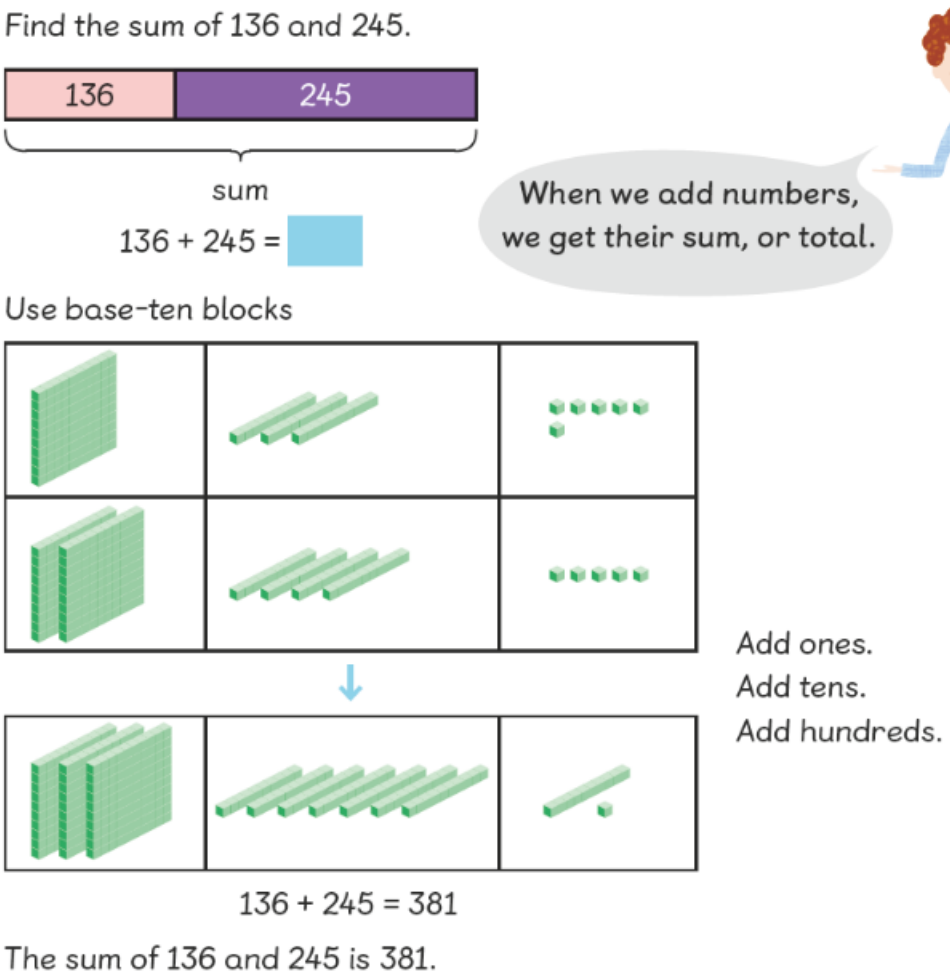


Initially addition will not require Renaming (previous referred to as carrying)

Adding Ones, Tens, Hundreds, …

Year 4

Children take previous understanding and apply it in contexts where more than one strategy is needed.



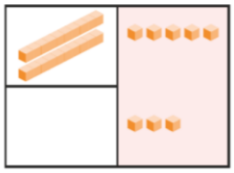
Layout and working frames are used to prepare the children for the formal method of column addition.

Year 1

Year 2

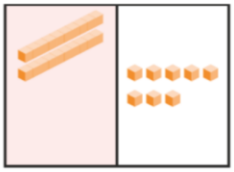
Adding Ones

Step 1 Add the ones.
5 ones + 3 ones = 8 ones



	tens	ones
	2	5
+		3
		8

Step 2 Add the tens.

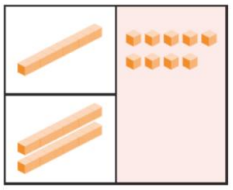


	tens	ones
	2	5
+		3
	2	8

25 + 3 = 28

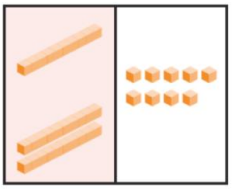
Adding Whole Tens

Step 1 Add the ones.



	tens	ones
	1	9
+	2	0
		9

Step 2 Add the tens.
1 ten + 2 tens = 3 tens



	tens	ones
	1	9
+	2	0
	3	9

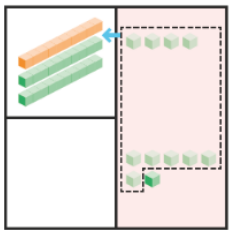
19 + 20 = 39

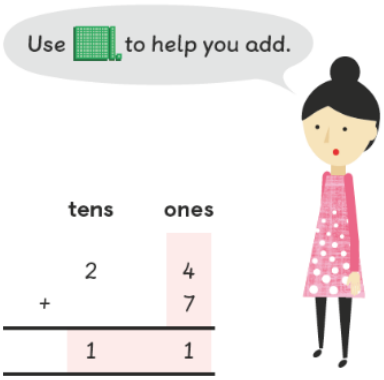
Children start to learn the formal method for addition by focusing on changing one place value column at a time adding either ones or whole tens.

Adding with Renaming

Add 24 and 7.

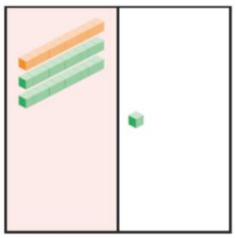
Step 1 Add the ones.
4 ones + 7 ones = 11 ones
Regroup the ones.
11 ones = 1 ten and 1 one





	tens	ones
	2	4
+		7
	1	1

Step 2 Add the tens.
1 ten + 2 tens = 3 tens



	tens	ones
	2	4
+		7
	1	1
+	2	0
	3	1

24 + 7 = 31

Renaming is required when the digit sum is 10 or larger. In this example the digit sum of the ones column is 11 so 11 ones is renamed 1 ten and 1 one.

This is an expanded method showing the total of the ones and tens before calculating the final total.

Formal Methods

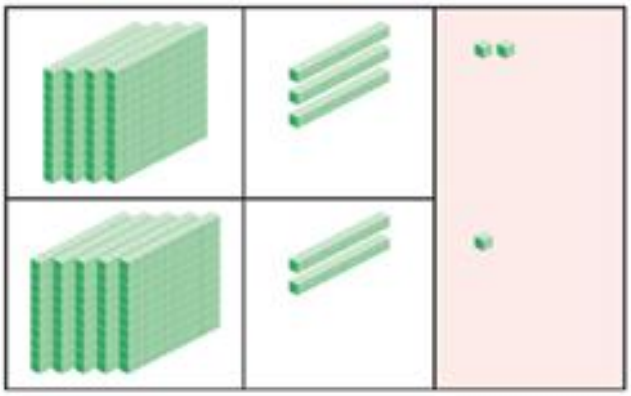
Year 3

Adding with no Renaming

Add 432 and 521.

Step 1 Add the ones.

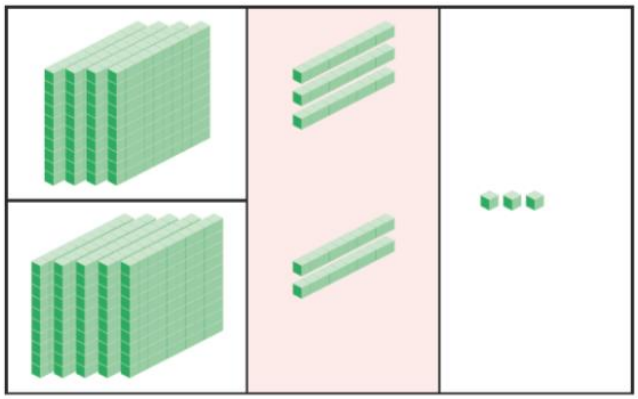
2 ones + 1 one = 3 ones



	h	t	o
	4	3	2
+	5	2	1
			3

Step 2 Add the tens.

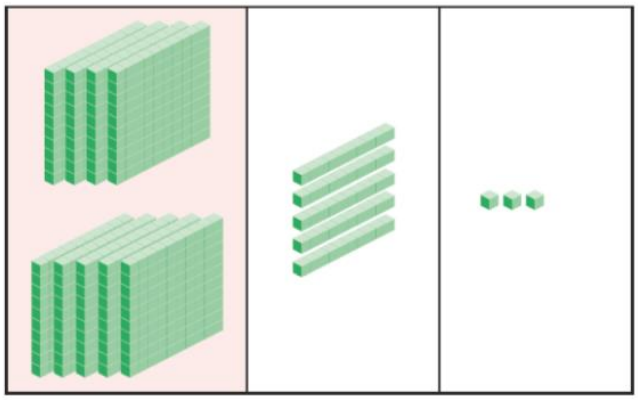
3 tens + 2 tens = 5 tens



	h	t	o
	4	3	2
+	5	2	1
		5	3

Step 3 Add the hundreds.

4 hundreds + 5 hundreds = 9 hundreds



	h	t	o
	4	3	2
+	5	2	1
	9	5	3

432 + 521 = 953

There are 953 flowers altogether.

Year 3

Adding with Renaming

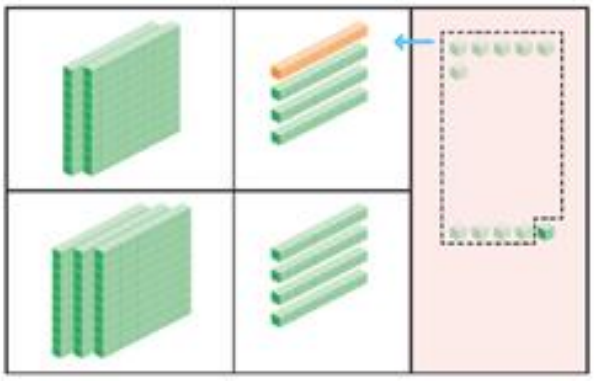
Add 236 and 345.

Step 1 Add the ones.

6 ones + 5 ones = 11 ones

Regroup the ones.

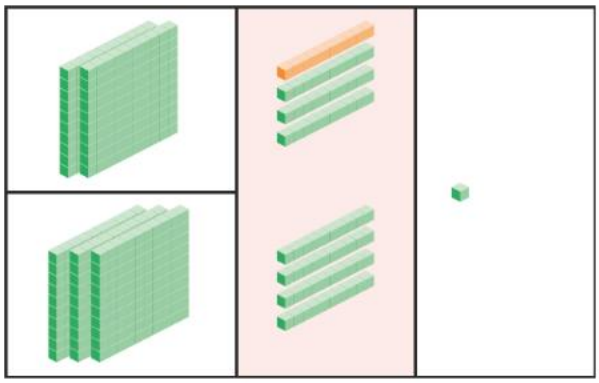
11 ones = 1 ten + 1 one



	h	t	o
		1	
	2	3	6
+	3	4	5
			1

Step 2 Add the tens.

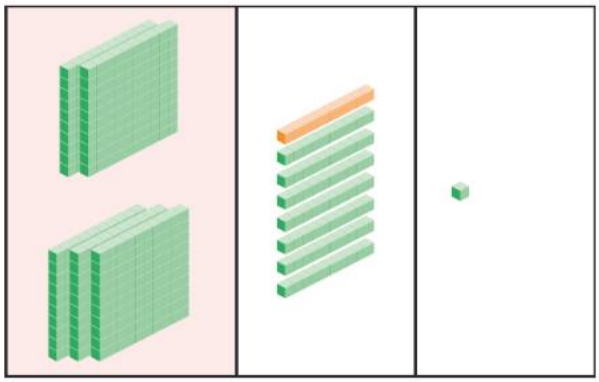
1 ten + 3 tens + 4 tens = 8 tens



	h	t	o
		1	
	2	3	6
+	3	4	5
		8	1

Step 3 Add the hundreds.

2 hundreds + 3 hundreds = 5 hundreds



	h	t	o
		1	
	2	3	6
+	3	4	5
	5	8	1

236 + 345 = 581

This is a compact method and doesn't show each sum on its own lines. Instead where a digit sum is 10 or more the renaming occurs at the start of the appropriate column.

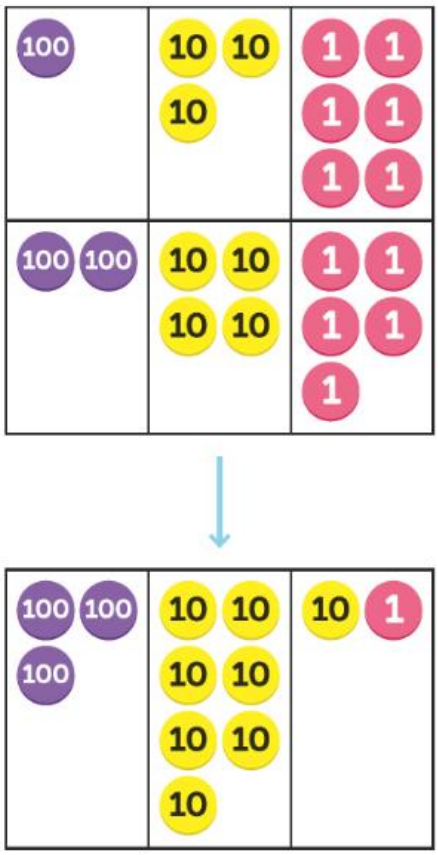
In this example there is only renaming of 11 ones to 1 ten and 1 one.

Formal Methods

Year 4

Expanded Method

Use number discs



136 + 245 = ☐

	1	3	6
+	2	4	5

Here place value counters or number discs provide concrete examples for the children to develop their understanding.

Add ones.
Add tens.
Add hundreds.

	1	3	6	
+	2	4	5	
		1	1	add ones
		7	0	add tens
+	3	0	0	add hundreds
	3	8	1	sum

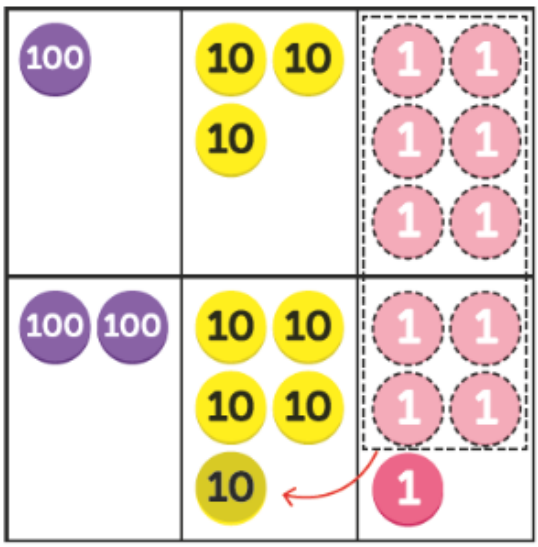
expanded

Compact Method

Use column addition

	1	1 3	6
+	2	4	5
	☐	☐	1

The sum is ☐.

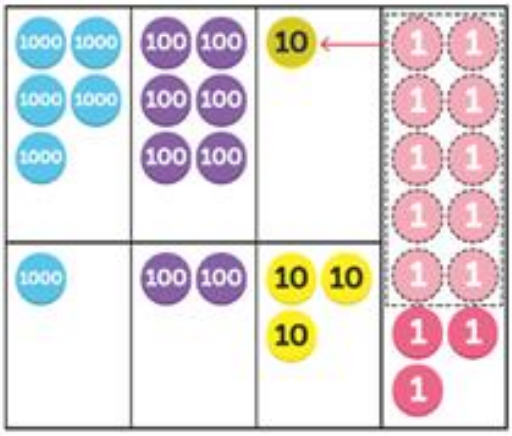


Year 4

Find the sum of 5608 and 1235.

Step 1 Add the ones. 8 ones + 5 ones = 13 ones

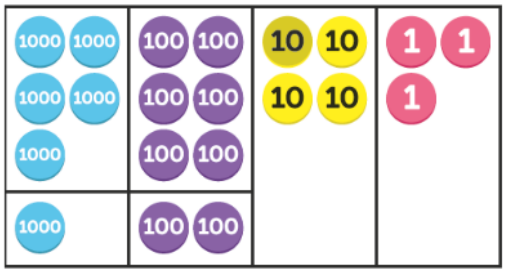
Rename the ones. 13 ones = 1 ten and 3 ones



$$\begin{array}{r} {\scriptstyle 1} \\ 5\ 6\ 0\ 8 \\ +\ 1\ 2\ 3\ 5 \\ \hline \\ 3 \end{array}$$

Step 2 Add the tens.

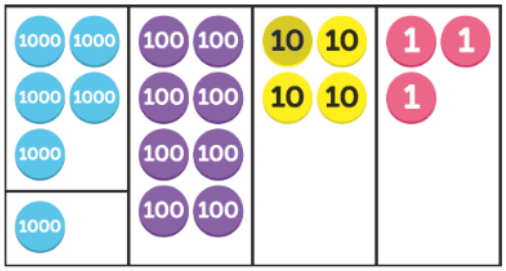
0 tens + 3 tens + 1 ten = 4 tens



$$\begin{array}{r} {\scriptstyle 1} \\ 5\ 6\ 0\ 8 \\ +\ 1\ 2\ 3\ 5 \\ \hline \\ 4\ 3 \end{array}$$

Step 3 Add the hundreds.

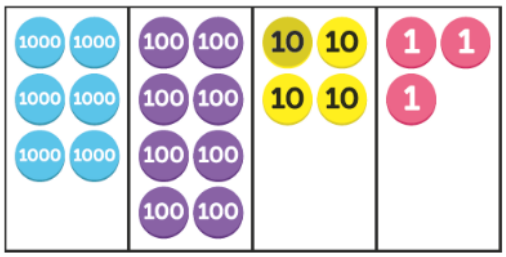
6 hundreds + 2 hundreds = 8 hundreds



$$\begin{array}{r} {\scriptstyle 1} \\ 5\ 6\ 0\ 8 \\ +\ 1\ 2\ 3\ 5 \\ \hline \\ 8\ 4\ 3 \end{array}$$

Step 4 Add the thousands.

5 thousands + 1 thousand = 6 thousands



$$\begin{array}{r} {\scriptstyle 1} \\ 5\ 6\ 0\ 8 \\ +\ 1\ 2\ 3\ 5 \\ \hline 6\ 8\ 4\ 3 \end{array}$$

5608 + 1235 = 6843

6843 tickets were sold.

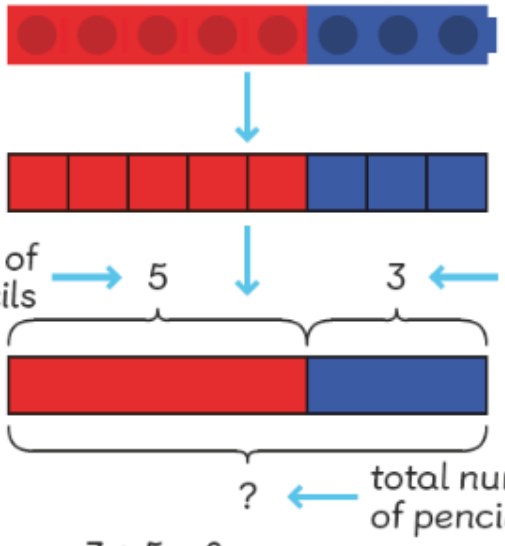
In Year 4 children will also work with numbers larger than 1,000 with additions that could require Renaming in multiple place value columns.

Bar Models

Year 3

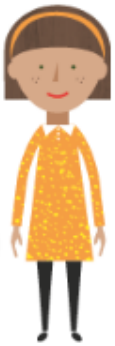
Children will be shown bar models on a regular basis as a pictorial representation of the problem. While this does not solve the problem, it will help the children understand what is being asked so that they can choose a method.

1 Use  to show the number of pencils.



$5 + 3 = 8$ or $3 + 5 = 8$

There are 8 pencils altogether.



Draw bars to show each number.

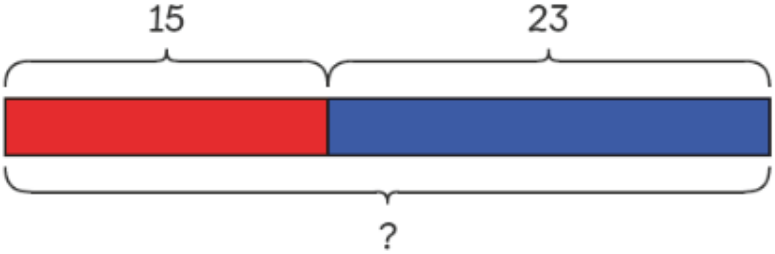
2

has 15 pencils.

has 23 pencils.

There is not enough space to use  !

How many pencils do they have altogether?



$15 + 23 = 38$

They have 38 pencils altogether.

	t	o
	1	5
+	2	3
	3	8